COURSE

OF

INSTRUCTION,

ORIGINALLY

COMPOSED FOR THE USE

OF THE

Royal Engineer Department.

BY

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FIELD WORKS.

VOLUME I.

Containing Practical Geometry and the Principles of Plan Drawing.

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PREFACE.

To those who have received a regular mathematical education, a new book upon Practical Geometry, which forms the principal part of the contents of this volume, may appear entirely superfluous; as the subject is simple in itself, as it has already been illustrated by a great number of writers, and more particularly as no new problem is now offered to the public.* I shall therefore state the reasons, which have induced me to compose and publish a work of this kind; and as they cannot otherwise be clearly understood, I must commence by making some observations of a military nature.

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* The same remark will apply to almost every other book upon this subject, which has been published for many years back; it being rare to find a problem in any of the common treatises of Practical Geometry, which has not previously appeared in a great number of other authors. This is the case with the whole of the contents of the first part of this volume, three useful problems only excepted; namely, the third method of describing an ellipse, the second method of describing a parabola, and the method of describing an oval. These I have taken from one of Mr. P. Nicholson's architectural works; and they are not (at least to my knowledge) to be found in any other writer upon Practical Geometry.

As far as regards the concluding part of this volume, which treats of the Principles of Plan Drawing, it will be found more original. I have not seen these principles clearly explained by any former author: indeed, it is not usual to teach them at all, which to learners is a great disadvantage; for as they are left to work at random, and to form rules for themselves from practice, they are of course frequently liable to error.

In the imperfect state of the art of [illegible] Engineers were at first a corps of officers solely, without any troops or even stores under their own immediate command or charge; but who, when occasion required, demanded men from the infantry, and stores from the Artillery or Commissariat Departments.

There are most serious evils and inconveniencies inseparable from the above system, which could not fail to be discovered in course of time, so that in all armies, it has ultimately been found necessary to attach a permanent body of non-commissioned officers and soldiers to the Engineer Department; and the experience of modern warfare has fully proved, that however great the science or talents of any officers of Engineers may be, their exertions in the field are in all cases much crippled, and must often be liable to failure, unless supported by a proportional degree of zeal, knowledge, and ability, on the part of the men, who act under their immediate orders.

In the British service, this has been more felt during the present, than in any of our former wars, owing to the increased scale of our late military efforts; and it has therefore become an acknowledged object, of the greatest public importance, to discipline, train, and instruct the non-commissioned officers and soldiers of the Royal Engineer department, who have lately been distinguished by the title of Royal Sappers and Miners, in lieu of their former less martial appellation of Royal Military Artificers.*

In the year 1811, having the command of the Plymouth company of Royal Military Artificers, I was induced to embrace the opportunity which the conveniency of a garrison life afforded, in order to ascertain by experiment the best and most practicable mode of improving the corps in general in point of knowledge. It had long been considered desirable, that the non-commissioned officers and soldiers should be able to understand the nature of a rough sketch, plan or section. To this object my attention was consequently directed, but I soon discovered that the common methods of teaching Practical Geometry and Plan Drawing, were by no means calculated for the purpose in view.

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* They were at one time called the Corps of Royal Military Artificers and Labourers. Subsequently, the enlistment of labourers was discontinued.

After instructing one or two individuals of the most promising abilities, books and manuscripts were put into their hands, and they were employed to teach the others; but although they themselves perfectly understood what they had learned, it was found that they were unable to communicate the knowledge which they had acquired. The reason of this is, that the common books of Practical Geometry, &c. leave too much to the discretion of the master; so that no man, however well qualified himself in this simple branch of learning, is capable of instructing a number of other men properly, unless he possesses a superior degree of judgement, a tolerably good education, and some experience in the art of teaching.

To have carried on a general system of the proposed species of instruction according to any of the books in common use, would therefore have required an establishment of regular mathematical masters. But it must be evident, that the difficulty of finding qualified persons willing to undertake this office, added to the liberal salary, which men of education must have been entitled to, would have operated as almost insuperable objections to such an establishment.

To surmount these difficulties; to lay down a Course of Instruction suited to the most untutored minds, and capable of being conducted by any man of good abilities, no matter how illiterate or ignorant in other respects; in short, to establish a System of Instruction, which might be perpetuated like the drill of recruits, by the exertions of steady non-commissioned officers employed as teachers, without the necessity of calling in the assistance of scientific masters of any kind: these are the points which I had in view in first commencing the present work; and the success has fully equalled my expectations.*

After the practicability of the System of Instruction had been sufficiently ascertained by the improvement of the Plymouth company, it was, by order of Lieutenant General Mann, submitted to a Committee of senior officers of the Corps of Royal Engineers, in the month of March, 1812, and having been honoured by their approbation, it was soon after sanctioned by the authority of the Master General of the Ordnance, and has since been conducted on a much greater scale at Chatham.†

The Methods of Teaching are fully detailed in the course of the work. It will be perceived, that they are similar in principle to those which have lately been introduced into this country for the education of the poor, by Dr. Bell and Mr. Lancaster. Indeed, there is no other principle, by which one Teacher can do justice to any great number of pupils at the same time.

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* Before the Course was gradually reduced to its present state, which renders the active interference of any officers of Engineers unnecessary; my first experiments at Plymouth, as it may easily be conceived, were attended with some trouble, and would have been still more laborious, but for the able assistance of Lieutenant Machell, who exerted himself with great zeal and perseverance to forward the object in view.

† At an Establishment which has been instituted for the purpose of training the Corps of Royal Sappers and Miners. But the study now alluded to, of course occupies only a very small portion of their time, which is chiefly employed in exercises and operations of a military or practical nature, calculated to prepare them for their duties in the field.

This volume having been composed solely for the special purpose above stated, I did not at first intend to publish it; but have since been induced to take this step from the following considerations.

In the first place, a more general diffusion of some knowledge of Practical Geometry amongst the lower classes, in this country, might be of public benefit; inasmuch as it would tend greatly to the perfection of the arts, by increasing the ingenuity of working mechanics, such as masons, carpenters, &c. without whose aid the designs of the architect or civil engineer cannot be executed. If this should be considered a desirable object, the present book offers the only economical and therefore it may be said the only practicable mode of carrying it into effect.*

Secondly: owing to the prevailing system in Great Britain, it often happens, that young gentlemen who have received a good classical education, are allowed to enter the world totally uninstructed in any branch of mathematics, excepting the common rules of arithmetic. But there are few situations in life, and very few active professions, in which some knowledge of Geometry, and of the nature of Plans, is not highly useful: in many it is absolutely necessary. To those, however, who have not had the advantage of early instruction in this science, the door of improvement is almost always shut; for the studies of Geometry, &c. in the manner in which they are treated in the common mathematical books, cannot be attained without the assistance of a master; and upon the whole, they generally prove by far too formidable a task to persons who have once launched forth into active life.

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* A person of very moderate acquirements and pretensions may serve for a Teacher, and it is not necessary for the learners to be provided with books, one copy in the hands of the Teacher being sufficient.

To such persons, but particularly to officers of the army, who may not have had an early mathematical education, the present volume may therefore prove useful; it having been reduced to principles of so much simplicity, that a master may be dispensed with; and the various operations contained in it may even be understood by inspection, without that effort of mind which deserves the name of study.

Another volume, more especially intended for the use of officers, is proposed to be published at some future opportunity, if circumstances will permit, in which the principles herein laid down will be practically applied to the construction of the Plans and Sections of a regular Fortress; as also to general and detailed plans, &c. of the operations of a siege. This will, in short, comprehend the substance of what is usually taught as the Elementary Part of Fortification. To simplify that important study as much as possible, by freeing it from those multiplied details, superfluous discussions, and even mathematical theorems and algebraical calculations which are usually mixed with it, and tend greatly to perplex the reader, will be my principal object in the proposed volume; which, I conceive, if successfully executed, may be of some public benefit, particularly as there is a great scarcity of English books upon the subject.

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COURSE
OF
PRACTICAL GEOMETRY,
&c.

In order to go through this Course, the learners must be provided with the following instruments.

1. *A pair of compasses having one shifting leg; also a pencil-point, and a short drawing pen or ink-point, occasionally to be used in lieu of the moveable leg.*

2. *A flat ruler, and a wooden right-angled triangle. The ruler may be about one foot long. The triangle should have a small round hole through its middle: those two sides of it, which form the right angle, may be made equal in length, and may each be about six inches long.*

3. *A slate and pencils. It will be convenient to have the slates rather larger than those commonly used; but they must not be framed.*

Besides a common set of drawing instruments, the Teacher must provide himself with a very large pair of compasses, one leg of which must be fitted so as to receive a piece of common chalk. He must also have a large triangle, and some long rulers of different lengths. These instruments are intended for drawing with chalk upon a large board painted black, the use of which will be more particularly described in the proper place. A spunge and water must always be kept ready for cleaning the board.

Every thing necessary being provided, the Teacher will see that the learners are seated in proper order, with their slates and instru-

*ments before them. He will then place himself in front of them, and read as follows.**

Geometry is the groundwork of almost all the arts and sciences. Plan drawing, and Modelling in particular are entirely founded upon the practical part of geometry.

Drawing is not only essentially necessary to men of scientific professions, such as engineers, architects, &c.; but some knowledge of it is even required amongst the workmen, who act under their directions. For instance, if a foreman of civil carpenters or masons, or a non-commissioned officer in the royal engineer department cannot work from a plan or model, he is not fit for his situation. Almost every ingenious artificer, in following his trade, must acquire some notion of the nature of a plan, and may daily practise some rule of geometry, although he may not be aware of it; because all the mechanical arts are founded upon geometrical principles: but the kind of knowledge, that is thus obtained by mere practice without education, must always be very imperfect.†

The following Course of Practical Geometry has been reduced to a system of so much simplicity, that a man having the book before him may even teach himself without a master. You will therefore find little difficulty in learning it, if you pay proper attention. It contains a number of the most useful rules, which will give you a sufficient insight into the principles of plan drawing.

After going through this course, such of you as understand arithmetic will also be qualified to begin, and may easily learn, Mensuration; which is a very desirable branch of knowledge to men of all professions, but is above all most particularly useful and necessary to persons employed in the management of works.

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* Those parts of the following Course, which are printed in italics, being intended as directions for the Teacher only, are not to be read aloud.

† These preliminary observations being calculated chiefly for the use of the non-commissioned officers and soldiers of the Engineer department, any Teacher who has pupils of a different description may omit such part of them as he thinks proper.

Having made these observations, we shall now commence with our practical geometry.

Write the words PRACTICAL GEOMETRY.

After giving this order, the Teacher will write these words himself, in large characters, with chalk, upon the abovementioned board, which must be placed in a convenient situation for the learners to copy from.

This board must of course be of a sufficient size to admit of every thing necessary being written or drawn upon it, on so large a scale as to be easily read or distinguished by the whole class. When the Teacher observes that all the learners have finished writing the words directed, he will examine and correct their respective performances, before he proceeds further. The same must be done after every subsequent order of a similar nature, but this caution will not be repeated again in the following instructions.

A definition in geometry, or in any other art, means an explanation of some term, peculiar to that art.

Write the words DEFINITION OR EXPLANATION.

To define any thing is to explain it.

Write the words TO DEFINE.

DEFINITION 1. A solid is that which has length, breadth, and thickness.

For instance a brick, a log of wood, or a lump of iron are solids.

Write the word SOLID.

DEF. 2. The boundaries of a solid are called superficies or surfaces.

Write the words SUPERFICIES OR SURFACE.

For instance the top of a brick is a superficies: the bottom of it is also a superficies; and so are the two ends, and the two sides of it. In short a superficies is merely a part of the outside of a solid.

A superficies has length and breadth only, but it is supposed to have no thickness.

For instance take the outside of a brick: it has length and breadth; but you cannot find any thickness without cutting into the substance of the brick, which would be going deeper than the outside.

A shadow gives a very just notion of a superficies or surface; for you may measure its length and breadth, but it has no thickness or substance.

DEF. 3. A plane superficies is that which is perfectly even; so that if you lay the edge of a ruler upon it in any direction, the ruler will touch it in every point.

For instance the outside of a marble slab properly polished is a plane superficies.

The Teacher must have a solid piece of wood, with six faces or sides, about the size of a brick or a little smaller; five of the sides of which must be planed quite smooth to represent plane superficies; the other must be uneven.

After reading the above definition, he will take a ruler and apply it to one of the smooth sides of his piece of wood in va-

rious directions, in order to illustrate the nature of a plane superficies.

A plane superficies is sometimes simply called a plane.

Write the words PLANE SUPERFICIES OR PLANE.

DEF. 4. A curved superficies means a crooked or uneven superficies; and is such as will not agree with the edge of a straight ruler, laid upon it in any direction.

For instance the top of a well-made new table is a plane superficies; but if the wood should get warped afterwards, it will become a curved superficies.

Here the Teacher will apply a ruler to one of the uneven sides of his piece of wood, in order to illustrate this definition.

Write the words CURVED SUPERFICIES.

DEF. 5. The boundaries of a superficies are called lines.

Write the word LINE.

A line has length only, but it is supposed to have no breadth nor thickness.

In order to illustrate this definition, the Teacher must produce a dark-coloured board, with a half-sheet of paper pasted upon it; the board being somewhat larger than the paper each way.

For instance let this half-sheet of paper represent a superficies: you see that it is bounded by four lines which are the top, the bottom, and the right and left sides of the paper.

You may measure the length of any of these lines, but you cannot find any breadth or thickness, unless you cut into the paper itself, which would be going deeper than the outside or boundary of it.

DEF. 6. The boundaries or extremities of a line are called points.

Write the word POINT.

A point being merely the end of a line is supposed to have neither length, breadth, nor thickness.

For instance one of the corners of this half-sheet of paper is a point, for it is the end of two lines; but it has no magnitude or size, because if it had, you might measure its length or breadth; but you cannot do that without cutting into the paper, or cutting off a piece of one of the lines that form the boundaries of the half sheet. This would be going deeper than the end of the line.

Magnitude and size being the same thing:

Write the words MAGNITUDE OR SIZE.

DEF. 7. A right line is a straight line, or one that lies even between its extreme points.

Write the words RIGHT LINE.

Draw a right line.

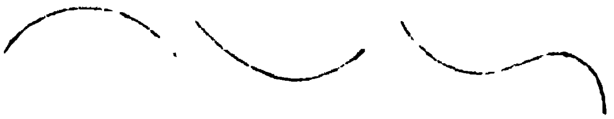
Here the Teacher will draw a right line upon the board, which the learners will copy: he will then examine and correct their performances before he proceeds further.

In like manner in the ensuing part of these instructions, whenever there is an order to draw any thing, the Teacher will recollect always to show the example himself; first executing upon the board whatever is required to be done; and afterwards examining the performances of the learners step by step. This caution will not be again repeated.

DEF. 8. A curved line is a crooked line, or one that does not lie even between its extreme points.

Write CURVED LINE.

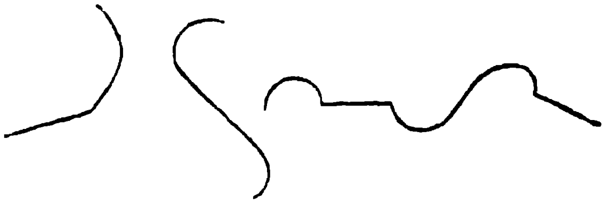
Draw some curved lines.



DEF. 9. A mixed line is partly right, and partly curved.

Write MIXED LINE.

Draw some mixed lines.



DEF. 10. Parallel lines are lines that are always at the same distance from each other, and will never meet, if you produce them ever so far.

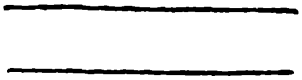
Write the words PARALLEL LINES.

Here the Teacher will draw two parallel right lines on the board.

You see that if I take my compasses and measure the distance between these two right lines in different parts it is always the same.

I shall now produce them or make them longer.

The Teacher will produce his two parallel lines.



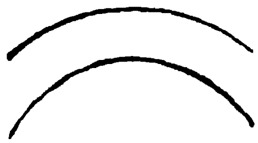
Now I shall measure the distance between the produced parts.

You see that it still remains the same; therefore these are parallel lines.

Write the words PRODUCE A LINE.

Curved lines may also be parallel to each other.

Here the Teacher will draw two parallel curved lines, and measure the distance between them, in like manner, at various parts, for the instruction of the learners.



When this is done, he will rub out his parallel lines.

In geometry any thing which a learner is required to do practically, as well as to understand, is called a problem.

Write the word PROBLEM.

I will now teach you the way to draw parallel right lines, which shall be our first problem.

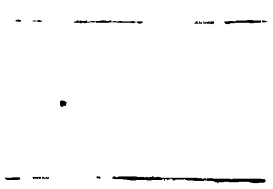
PROBLEM I.

THROUGH A GIVEN POINT TO DRAW A RIGHT LINE PARALLEL TO A GIVEN RIGHT LINE.

METHOD 1. By a pair of compasses and ruler.

Draw a right line to represent the given right line.

Mark a point above it to represent the given point.



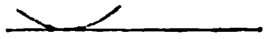
This is the manner in which I shall always mark my points upon the board.

In marking your points upon your slates you must not make them large as I shall do; but let them be as fine as possible. That I may know at first sight the position of these small points, you must make a small O or circle round them, except when they are marked in some part of a line. You will recollect, that this circle round a detached point is of no use, except to guide my eye, when I examine your slates.

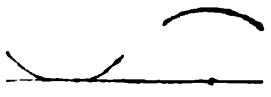
In the manner just directed mark a circle ⊙ round your given points, thus.

Measure the distance between this point, and the nearest part of the given line. The best way of trying whether you have got this distance correct, is to place one leg of your compasses in the given point, and afterwards to make a sweep downwards, with the other leg. If the sweep touches, but does not cut the given line, your distance is true.

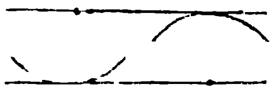
Make your sweep as directed. Recollect always in using your compasses, to hold them light in your hand, and not to press hard with either of the legs.



You will next place one leg of your compasses in some point, near the other end of the given line, and with the same opening make a sweep upwards.



Draw a right line through the given point, touching but not cutting the last made sweep.



Rub out your sweeps, and your problem is executed; for the two lines are parallel to each other, and one of them passes through the point that was required.

A sweep made with a pair of compasses is called an Arc.

Write the word ARC.

When a person makes a sweep with a pair of compasses, he is said to describe an arc.

Write the words DESCRIBE AN ARC.

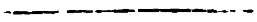
The point where one leg of the compasses is fixed, whilst you describe an arc with the other, is called the center.

Write the word CENTER.

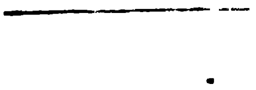
Recollect these terms in future.

You will now execute the same problem, with the given point below the given line.

Draw a new line to represent the given line.

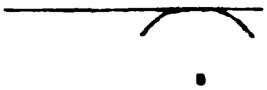


Mark a point below the line to represent the given point.

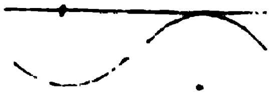


You are to draw a line through this point, parallel to your given line.

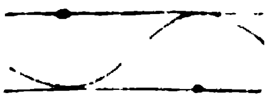
Measure in your compasses the distance between the given point and the nearest part of the given line, as you were before directed.



Then place one leg of your compasses in a more distant part of your given line, and describe an arc below it.



Draw a right line through the given point, touching, but not cutting your last described arc.



The problem is now executed; the two lines are parallel, and the last drawn line passes through the given point, as was required.

The Teacher will exercise the learners several times in performing this problem, making them place their point in different positions.

PROBLEM I.

THROUGH A GIVEN POINT TO DRAW A RIGHT LINE PARALLEL TO A GIVEN RIGHT LINE.

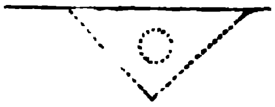
METHOD **2.** By a triangle and ruler without compasses.

Draw a right line to represent the given right line.

Mark a point above it to represent the given point.

You must now draw a right line parallel to the given right line, through the above point.

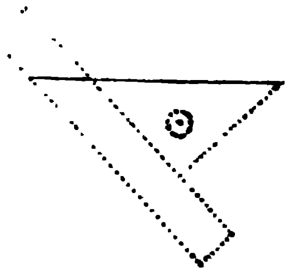
The long side of your triangles must be placed upon the given line, with the body of the triangle below the line.



Place triangles.

Here the Teacher will examine if the position of the triangles is correct.

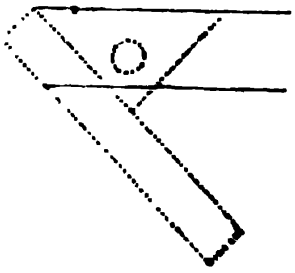
Keep your triangle steady with the right hand, whilst you apply the ruler with your left hand, to that short side of the triangle which is towards the left of your slate.



Place rulers.

Here the Teacher must examine the position of every man's ruler and triangle.

Keep your ruler steady with your left hand, and slide the triangle up with your right hand, till the long side of it meets the given point. Then draw a right line through the given point, by means of your triangle, and your problem is executed.



If the triangle is not large enough to draw the parallel as long as you wish, you may produce it afterwards by means of the common ruler.

The Teacher will then exercise the learners in repeating this problem, with new points, not only above, but below the given line.

When the given point is below the line, the only difference is, that the triangle must be slided downwards, after the ruler is placed.

When the learners are more expert, they must be made to take the ruler in the left hand, and the triangle in the right, and to place the triangle and ruler at the same time.

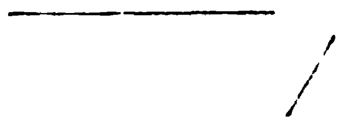
PROBLEM II.

AT A GIVEN DISTANCE TO DRAW A RIGHT LINE PARALLEL TO A GIVEN RIGHT LINE.

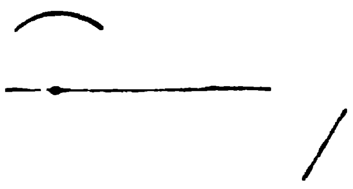
METHOD 1. By a pair of compasses and a ruler.

Draw a right line across your slate to represent the given right line.

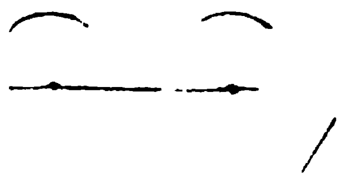
Draw a second right line at one corner of your slate, a good deal shorter than the former, to represent the given distance at which a parallel is to be drawn to the first line.

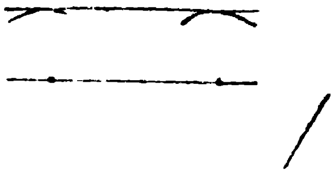


Take the length of the short line, or the given distance, in your compasses; and with any point near one end of the given line as a center, describe an arc, above *(or below)* it.



Then take some point near the other end of the given line, as a center, and describe a second arc, on the same side of it as the former.





Draw a right line touching both these arcs, and your problem is executed.

Rub out superfluous arcs.

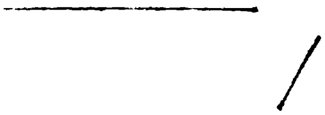
PROBLEM II.

AT A GIVEN DISTANCE TO DRAW A RIGHT LINE PARALLEL TO A GIVEN RIGHT LINE.

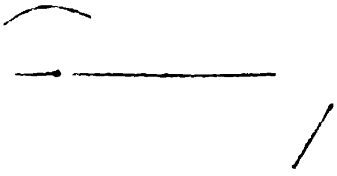
METHOD 2. By a triangle and ruler as well as a pair of compasses.

Rub out your former figure.

Draw a right line to represent the given right line.



Draw another right line to represent the given distance.



Take the given distance in your compasses, and from any point near one end of the given line, as a center, describe an arc above *(or below)* it.

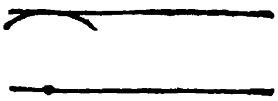
Then place the long side of your triangle, upon the given line, with the body of the triangle below it.

Take the triangle in your right hand; and apply your ruler to the left side of it.

Steady the ruler with your left hand, and slide the triangle with your right, till it meets that part of the arc, which is at the greatest distance from the given line.

After each of these orders the Teacher will examine the position of the triangles, &c.

Draw a line along the edge of the long side of your triangle, touching but not cutting the given arc.



This last drawn line will be parallel to your given right line, at the distance required.

The problem is therefore executed.

The Teacher will make the learners perform this problem as well as the former several times over. He will also recollect to observe the same rule in respect to every succeeding problem, making them do it repeatedly until he thinks they are perfect.

We shall now proceed with some more definitions.

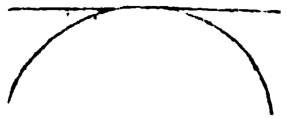
DEF. 11. Any line which touches a curve or an arc, but does not cut it, is called a tangent.

You have already drawn several tangents.

Write the word TANGENT.

Draw a curve.

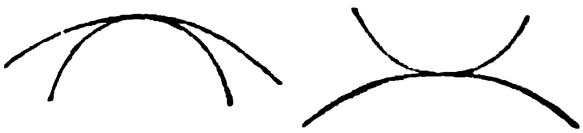
Draw a right line as a tangent to the curve through any point you please.



Rub out your figure.

One curved line may also touch or be a tangent to another.

Draw two curved lines touching each other.

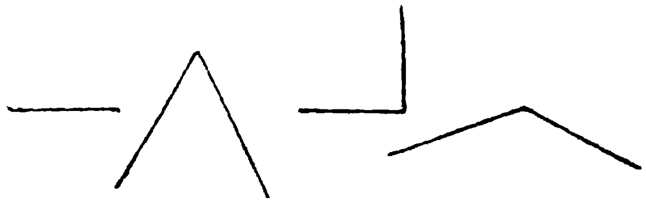


The Teacher may choose either or both of these kinds of curved tangents.

DEF. 12. When two lines incline towards each other, so as to meet in a point, they are said to form an angle.

Write the word ANGLE.

Draw some angles.



An angle is measured not by the length of the two lines that form it, but by the greatness of their opening, or as workmen would call it, by their splay, or bevel.

DEF. 13. When one line standing upon another, does not incline or lean more to one side than to the other, it forms two angles, exactly equal to each other, which are called right angles.

The Teacher must draw this upon the board.

Write the words RIGHT ANGLES.

DEF. 14. When two lines form a right angle, or are at right angles to each other, the one line is said to be perpendicular to the other.

Write the word PERPENDICULAR.

I shall now teach you the method of drawing perpendiculars.

PROBLEM III.

FROM A GIVEN POINT IN A GIVEN RIGHT LINE, TO RAISE OR DROP A PERPENDICULAR, BY A RULER AND COMPASSES.

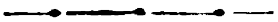
METHOD 1. When the point is supposed to be near the middle of the given line.

Draw a right line to represent the given right line.

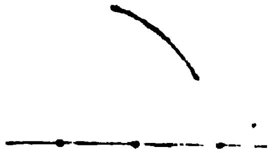
Mark a point near the middle of it, to represent the given point.

We shall first raise our perpendicular.

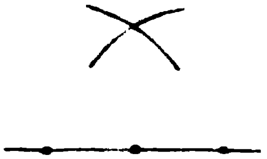
Place one leg of your compasses in the given point, and with any opening you please, mark new points upon the given line, on each side of it, with the other leg.



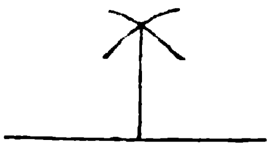
Take a greater opening in your compasses, and from one of the new points, as a center, describe an arc above the given line.



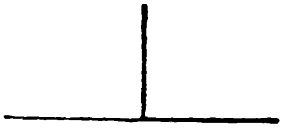
With the same opening of your compasses, from the second new point, as a center, describe another arc, cutting the former.



From the point where these arcs cut each other, draw a right line to the given point, in the given line.



Rub out superfluous arcs.



The last drawn line is perpendicular, to the given line, and it is raised from the given point. Your problem is therefore executed.

The angles on each side of your perpendicular, are equal to each other, and are right angles, as was before explained.

When two arcs or two lines cut each other, they are said to intersect each other.

An arc may also cut, or intersect, a line.

Write the word INTERSECT.

The point where lines or arcs cut each other, is called the point of intersection.

Write the words POINT OF INTERSECTION.
Recollect these terms in future.

Rub out your figure.

The Teacher will then make the learners drop a perpendicular, from a given point in a given line, which is done exactly by the same method, only that the two arcs must intersect each other, below the given line.

PROBLEM III.

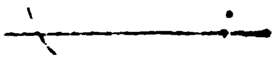
FROM A GIVEN POINT IN A GIVEN RIGHT LINE TO RAISE OR DROP A PERPENDICULAR, BY A RULER AND COMPASSES.

METHOD 2. When the point is near the end of the line.

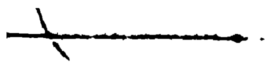
Draw a right line, to represent the given right line; and mark a point in it, near one end of it, to represent the given point.

Mark another point above the given line, not exactly over the given point, but more towards the middle of the line.

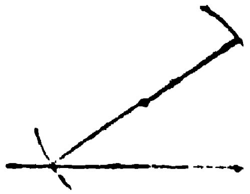
Take in your compasses the exact distance, between the two points, and from the point without the line, as a center, describe a small arc, intersecting the given line in another point.



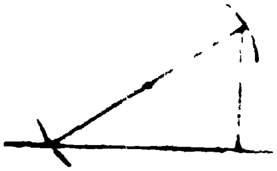
From the point without the line, as a center, with the same opening of your compasses, describe another arc, exactly opposite to the former.



From the point where the first arc intersects the given line, draw a right line through your center, and produce it till it cuts the second arc.



From the point of intersection on the second arc, draw a right line to the given point in the given line.



The line which you drew last is the perpendicular required. Consequently the problem is executed.

Rub out superfluous or unnecessary lines, points and arcs; leaving only the given line and the perpendicular.

When a perpendicular is to be dropped, it is done exactly in the same manner, only that the point without the line must be placed below it.

After reading this observation, the Teacher will cause the learners to drop perpendiculars accordingly.

PROBLEM IV.

FROM A GIVEN POINT TO DRAW A PERPENDICULAR TO A GIVEN LINE BY A RULER AND COMPASSES.

METHOD 1. When the given point is supposed to be nearly opposite to the middle of the line.

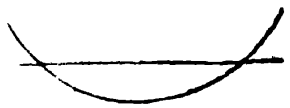
Draw a right line to represent the given line.

Mark a point above below the line nearly opposite to the middle of it, to represent the given point.

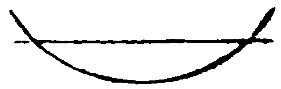
The Teacher will first cause the learners to mark the point above the line.

From this point, you are now required to raise drop a perpendicular to the given line.

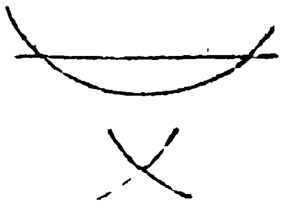
From the given point as a center, with any convenient opening of your compasses, describe an arc, intersecting the given line, in two places.



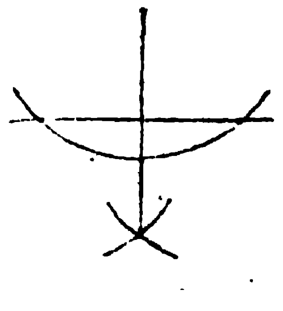
From one of these points of intersection, as center, with the same or any other convenient opening of your compasses, describe an arc on the opposite side of the line.

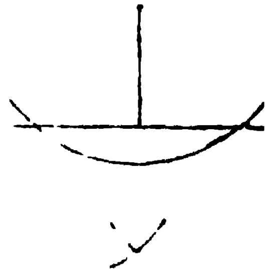


From the second point of intersection, as a center, with the same opening, describe an arc, intersecting the last arc.

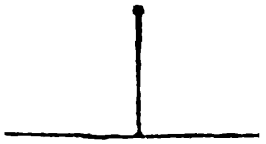


From the given point draw a right line, to the point of intersection, of the two arcs.





Rub out that part of this last drawn right line, which passes beyond the given line, and the remaining part of it will be the perpendicular required.



Rub out also superfluous arcs.

Your problem is now executed.

When the point, from whence the perpendicular is to be drawn, is placed below the given line, the problem will be performed exactly by the same rule.

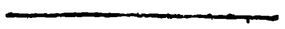
Here the Teacher will cause the learners to raise a perpendicular to a given line from a given point below the line, according to this method.

PROBLEM IV.

FROM A GIVEN POINT TO DRAW A PERPENDICULAR TO A GIVEN LINE, BY A RULER AND COMPASSES.

METHOD 2. When the point is nearly opposite to the end of the line.

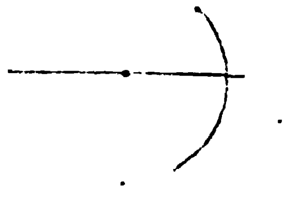
Draw a right line to represent the given line, and mark a point above below it, nearly opposite to one end of it, to represent the given point.



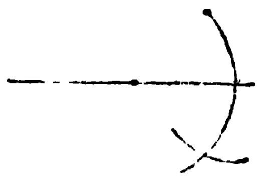
The Teacher will first cause the learners to place their point above the line.

Mark a point in the given line towards the middle of it.

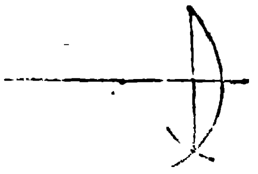
From the last marked point, as a center, with an opening, equal to the distance between the two points, describe an arc, which must commence at the given point, and cross the given line.



From the point where this arc intersects the given line, as a center, with an opening equal to the distance between this point and the given point, describe a second arc intersecting the former arc, on the opposite side of the line.



Draw a right line from the given point, to the point of intersection of the two arcs.



This will be perpendicular to the given line.

Rub out the superfluous part of this perpendicular.

Rub out also superfluous arcs; and your problem is executed.

When the point from whence the perpendicular is to be drawn is placed below the given line, the problem will be performed exactly by the same rule.

Here the Teacher will cause the learners to raise a perpendicular to a given line, from a given point below it, according to this method.

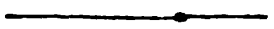
The drawing of perpendiculars, which formed the subject of the last two problems, may also be performed without a pair of compasses, by means of a triangle and ruler. The mode of doing this shall now be explained.

PROBLEM V.

FROM A GIVEN POINT TO RAISE OR DROP A PERPENDICULAR TO A GIVEN RIGHT LINE, BY MEANS OF A TRIANGLE AND RULER.

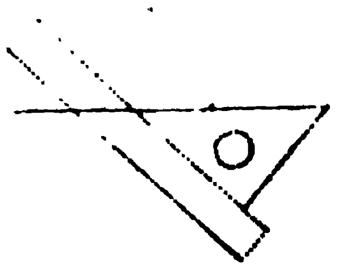
METHOD 1. Draw a right line to represent the given right
above
line, and mark a point in it, to represent the given point.
below

The Teacher will first make the learners mark their point in the given line.



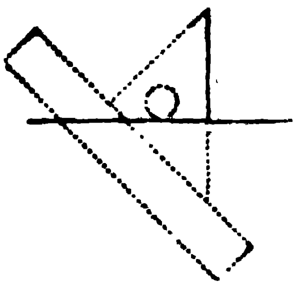
Place the long side of your triangle upon the given line, with the body of the triangle under it.

Hold the triangle firmly in your right hand, and with your left hand apply your ruler to that short side of the triangle, which is to the left of your slate, in the same manner that was before directed, in drawing parallel lines.



Steady your ruler with the left hand, and with your right hand turn the triangle round, till the other short side of it touches the ruler; the long side of the triangle will then become perpendicular to the given line.

Slide your triangle upon the ruler, till the long side agrees with the given point; then draw a right line from the point to the given right line.

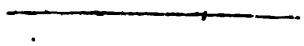


This last drawn right line will be the perpendicular required.

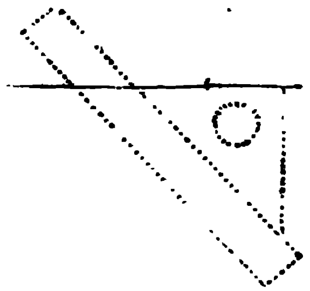
Rub out your figure.

METHOD 2. The same problem may be performed without turning your triangle.

Draw the given right line and the given point as before.

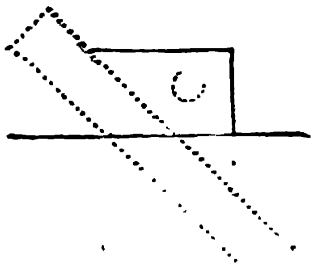


Place one of the short sides of your triangle upon the given line, and let the end of it be a good deal beyond the given point.



Then apply the ruler to the long side of your triangle, and steady the ruler.

Slide your triangle up the ruler, till the other short side of it meets the given point.



Along this last-mentioned side of your triangle, draw a right line from the given point to the given line.

This last drawn right line is the perpendicular required.

The Teacher will make the learners perform this problem repeatedly, placing their given point in various positions above, and below, as well as in the given line; the same rule applies to every case.

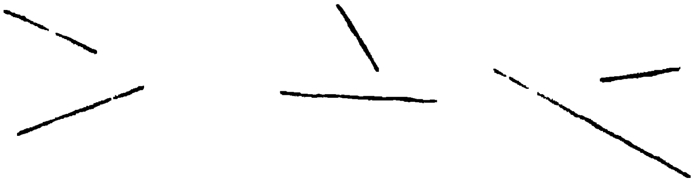
The learners should also know how to use the triangle and ruler, with either hand; as it may not always be most convenient to apply the ruler to the left side of the triangle.

We shall now return to our Definitions.

DEF. 15. Any line, that is neither parallel nor perpendicular to another, is said to be oblique to it.

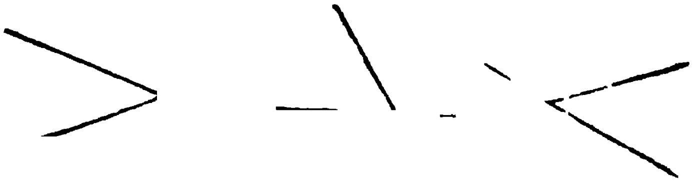
Write the words OBLIQUE LINES.

Draw some oblique lines.



Oblique lines if produced will meet and form an angle.

Produce your oblique lines till they meet, or as far as your slate will permit.



DEF. 16. Any angle, that is not a right angle, is called an oblique angle.

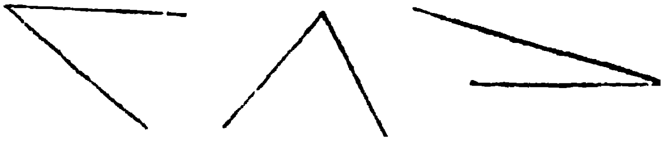
Write OBLIQUE ANGLES.

Oblique angles are of two kinds.

DEF. 17. An oblique angle which is less than a right angle, is called an acute angle.

Write ACUTE ANGLES.

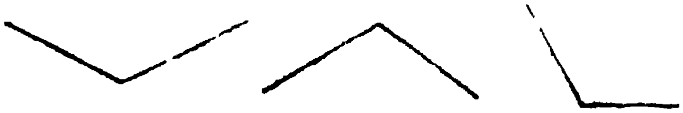
Draw some acute angles.



DEF. 18. Any oblique angle which is greater than a right angle, is called an obtuse angle.

Write OBTUSE ANGLES.

Draw some obtuse angles.



Having defined the nature of lines and angles, we shall now proceed to the various kinds of plane superficies.

A plane superficies was before defined.

It was stated that a superficies merely meant the surface of any solid, and that the surface or outside of any thing has a shape or figure, but no substance.

A superficies is therefore very often called a figure, because it is a figure and nothing more, being in short exactly like a shadow, as was before explained.

Write PLANE SUPERFICIES OR PLANE FIGURE.

A plane figure may either be bounded by right lines, or by curved lines, or both.

DEF. 19. A plane figure bounded by right lines is called a right-lined, or rectilinear figure.

Write the words RIGHT-LINED OR RECTILINEAR FIGURE.

DEF. 20. A plane figure bounded by curve lines is called a curve-lined or curvilinear figure.

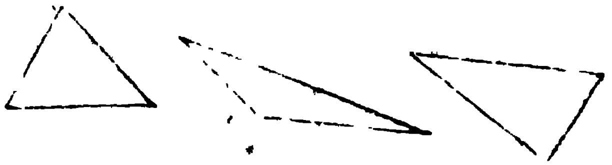
Write the words CURVE-LINED OR CURVILINEAR FIGURE.

A right-lined figure has exactly as many sides, as it has angles, the least number being three.

DEF. 21. A right-lined figure of three sides and angles is called a triangle.

Write A TRIANGLE.

Draw some triangles.



When a figure of several sides is mentioned without specifying particularly the nature of the lines which bound it, a right-lined figure is always implied.

DEF. 22. An equilateral triangle is that which has all its three sides equal.

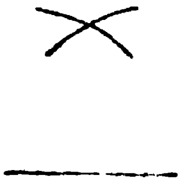
Write EQUILATERAL TRIANGLE.

PROBLEM VI.

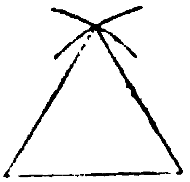
UPON A GIVEN RIGHT LINE, TO MAKE AN EQUILATERAL TRIANGLE.

Draw a right line to represent the given line.

Take the length of the line in your compasses, and from the two ends of it as centers, describe arcs intersecting each other above it.



From each end of the given line, draw a right line, to the point of intersection of the two arcs.



The triangle which you have just drawn has all its sides equal; and is therefore called an equilateral triangle: consequently your problem is executed.

Any other plane figure, which has all its sides equal, is also said to be equilateral.

Write AN EQUILATERAL FIGURE.

DEF. 23. The undermost side of a triangle, or of any figure, is called the base.

Write the words BASE OF A TRIANGLE OR OTHER FIGURE.

DEF. 24. When one side of a triangle is called the base, the other two are generally called the legs.

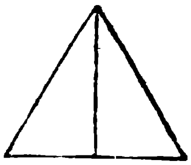
Write THE LEGS OF A TRIANGLE.

DEF. 25. The height of a triangle or other figure is called its altitude.

Write the words ALTITUDE OF A TRIANGLE, &c.

The altitude of a figure is measured by dropping a perpendicular, from the highest angle or top of it to the base.

Drop a perpendicular to show the altitude of your equilateral triangle.



DEF. 26. The top or highest angle of any figure is called the vertex.

Write the words VERTEX OF A TRIANGLE, &c.

DEF. 27. An isosceles triangle is one that has two of its sides equal.

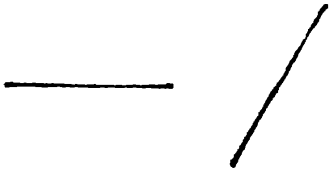
Write ISOSCELES TRIANGLE.

PROBLEM VII.

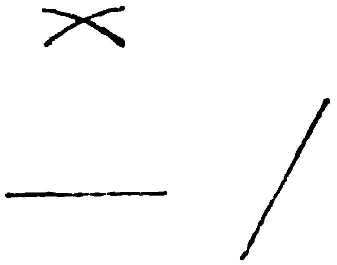
UPON A GIVEN LINE AS A BASE, TO DESCRIBE AN ISOSCELES TRIANGLE, WHOSE TWO OTHER SIDES SHALL EACH BE EQUAL TO ANOTHER GIVEN LINE.

Draw a right line to represent the base of your isosceles triangle.

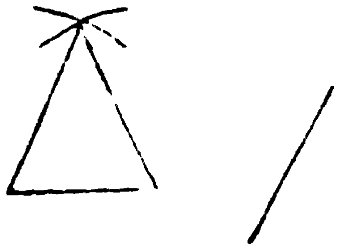
Draw another right line in a corner of your slate, to represent the given length of the two equal sides, observing only that you must make this second line more than one half of the former.



Take in your compasses the length of the second line, and from the ends of your base, as centers, describe two arcs intersecting each other above the base.



Draw right lines from the ends of your base, to the points of intersection of the two arcs.



The triangle which you have just drawn is an isosceles triangle; that is to say it has two sides equal to each other, and also equal to a given line; and its base is of a given length: consequently your problem is executed.

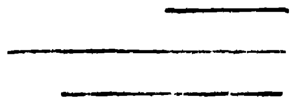
DEF. 28. A triangle which has all its three sides unequal, is called a scalene triangle.

Write SCALENE TRIANGLE.

PROBLEM VIII.

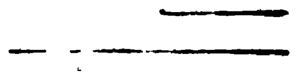
TO MAKE A TRIANGLE, WHOSE THREE SIDES SHALL BE EQUAL TO THREE GIVEN RIGHT LINES.

Draw three right lines of different lengths, at a corner of your slate, to represent the three given right lines; observing only that any two of these lines must be greater than the third, otherwise the problem will become impossible.



I shall now mark the line, which I intend to take for my base, with a point.

You will do the same.

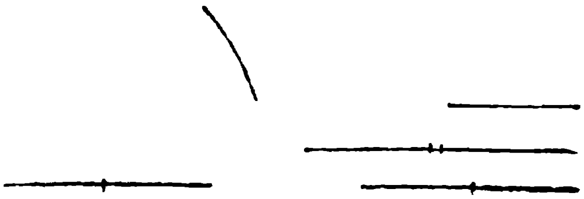


Draw a line near the middle of your slate, equal to the line which you have just marked; and let this be the base of your triangle.

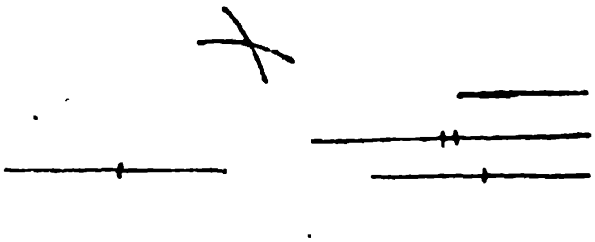
Mark it also with a point.

Mark another of your three given lines, with two points.

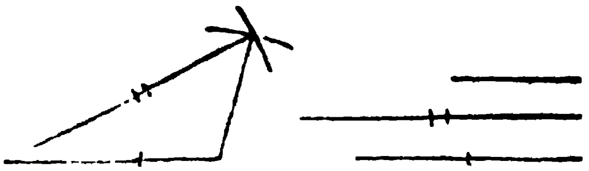
Take the length of this second line in your compasses, and from one end of your base, as a center, describe an arc over it.



Take the length of the third given line in your compasses, and from the other end of your base, as a center, describe a second arc intersecting the former.



Join both ends of your base to the point of intersection, and mark with two points that line which is drawn from the center of the first arc.



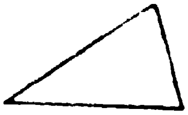
Your problem is now performed. You have made a triangle whose three sides are equal to the three given right lines, and those lines which are marked in the same way correspond or agree with each other.

Triangles have also different names according to the nature of their angles.

DEF. 29. A triangle which has its three angles all acute is called an acute angled triangle.

Write an ACUTE ANGLED TRIANGLE.

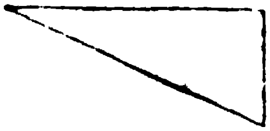
Draw an acute angled triangle.



DEF. 30. A triangle which has one right angle, is called a right angled triangle.

Write a RIGHT ANGLED TRIANGLE.

Draw a right angled triangle.

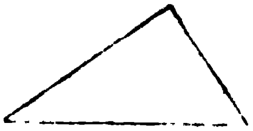


DEF. 31. The longest side of a right angled triangle is called the hypothenuse.

Write THE HYPOTHENUSE OF A RIGHT ANGLED TRIANGLE.

The hypothenuse of a right angled triangle, sometimes is the base of it.

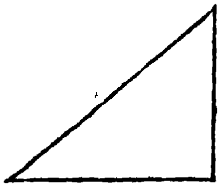
Draw a right angled triangle, having the hypothenuse for its base.



When the hypothenuse is the base, the other two sides of a right angled triangle are called the legs.

Sometimes one of the short sides of a right angled triangle is the base.

Draw a right angled triangle, having a short side for its base.



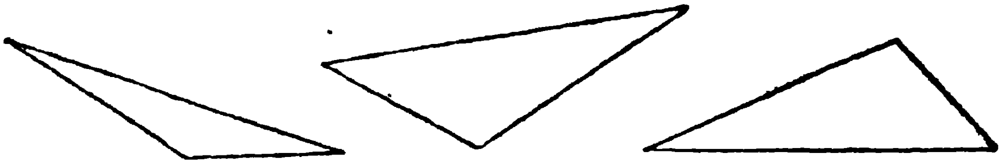
In this case the three sides are not called the hypothenuse and legs, but the hypothenuse, base, and perpendicular.

Write the words HYPOTHENUSE, BASE, AND PERPENDICULAR OF A RIGHT ANGLED TRIANGLE.

DEF. 32. A triangle which has one obtuse angle is called an obtuse angled triangle.

Write AN OBTUSE ANGLED TRIANGLE.

Draw some obtuse angled triangles.



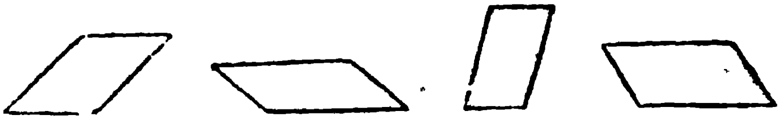
DEF. 33. Figures with four sides and angles are called quadrilaterals or quadrangles.

Write the words QUADRILATERAL or QUADRANGLE.

DEF. 34. A quadrilateral or four-sided figure, which has its two opposite pair of sides parallel to each other, is called a parallelogram.

Write the word PARALLELOGRAM.

Draw a parallelogram.

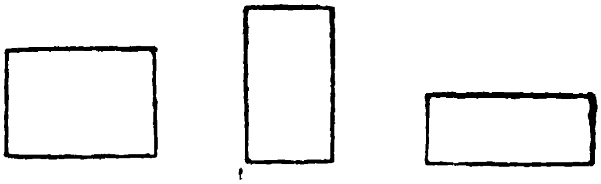


Here the Teacher will cause the learners, after he has examined their first parallelogram, to draw a second, a third, &c. all of different forms, such as are shown above; taking care to see that each figure is correct, before he allows them to begin another.

DEF. 35. A parallelogram which has all its angles right angles is called a rectangle.

Write the word RECTANGLE.

Draw a rectangle.



Here the Teacher will make the learners draw several rectangles, examining only one figure at a time, as was before directed.

DEF. 36. A rectangle which has all its sides equal is called a square.

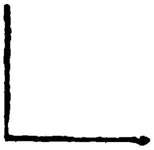
Write the word SQUARE.

PROBLEM IX.

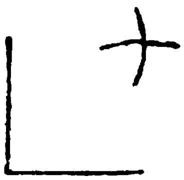
UPON A GIVEN LINE TO MAKE A SQUARE.

Draw a right line to represent the given line, and also the base of the square.

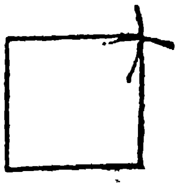
From one end of the base, raise a perpendicular, equal to it in length.



Take the length of the base in your compasses, and from the top of the perpendicular, and the farthest end of the base, as centers, describe arcs intersecting each other above the base.



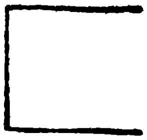
From the extremities of the base and perpendicular, draw right lines to the point of intersection of these two arcs: This will complete your square.



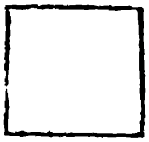
METHOD 2. Of making a square.

Rub out the two arcs and the two last drawn lines, leaving only the base and perpendicular.

From the top of the perpendicular, draw a line parallel to the base.



From the farthest extremity of the base, draw a line parallel to the perpendicular, and your square will be complete.



In Geometry, the word square signifies the four-sided figure which you have just drawn, and nothing else. It is common, however, amongst workmen to say, that two lines are square to each other, instead of perpendicular. This is a very improper term, which you will recollect never to use; unless you happen to meet with persons so very ignorant, as not to be able to understand you otherwise.

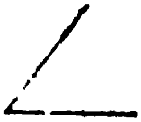
DEF. 37. A rhombus is an equilateral parallelogram, whose angles are oblique.

Write the word RHOMBUS.

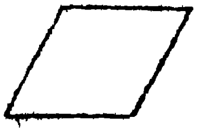
You will now proceed to draw a rhombus.

First draw a right line to represent the base of it.

Draw a second right line forming an oblique angle with the base, but equal to it in length.



From the top of this second line, draw a line parallel to the base; and from the other extremity of the base draw a line, parallel to your second line.



This will form a parallelogram, which has all its sides equal, and all its angles oblique: it is therefore a rhombus.

DEF. 38. An oblique angled parallelogram, whose sides are unequal, is called a rhomboid.

Write the word RHOMBOID.

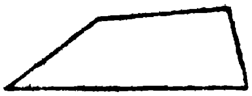
Draw a rhomboid.

Here the Teacher will make the learners draw some rhomboids. The figures of parallelograms given in Definition 34 *being all rhomboids, he may refer back to these if he finds it necessary.*

DEF. 39. A quadrilateral, which has none of its four sides parallel, is called a trapezium.

Write the word TRAPEZIUM.

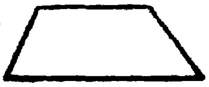
Draw a trapezium.

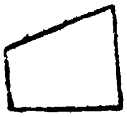


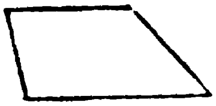
DEF. 40. A quadrilateral, which has one pair of parallel sides only, is called a trapezoid.

Write the word TRAPEZOID.

Draw a trapezoid.



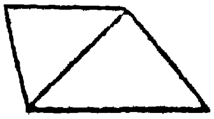




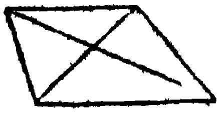
DEF. 41. A diagonal is a line drawn across a quadrilateral or other figure, between two opposite angles.

Write the word DIAGONAL.

Draw a diagonal across one of your trapezoids.



Draw the other diagonal.



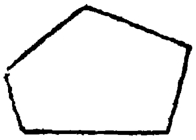
DEF. 42. Any figure with more than four sides, is called a polygon.

Write the word POLYGON.

DEF. 43. A polygon of five sides is called a pentagon.

Write the word PENTAGON.

Draw a pentagon.



DEF. 44. A figure with six sides is called a hexagon.

Write the word HEXAGON.

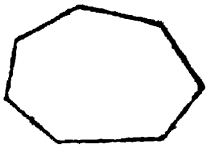
Draw a hexagon.

The Teacher must direct the learners to make the following figures as large nearly as their slates will hold.

DEF. 45. A figure with seven sides is called a heptagon.

Write the word HEPTAGON.

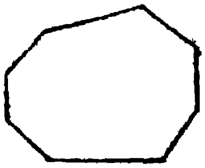
Draw a heptagon.



DEF. 46. A figure with eight sides is called an octagon.

Write the word OCTAGON.

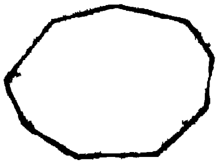
Draw an octagon.



DEF. 47. A figure with nine sides is called a nonagon.

Write the word NONAGON.

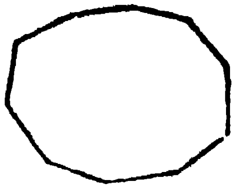
Draw a nonagon.



DEF. 48. A figure with ten sides is called a decagon.

Write the word DECAGON.

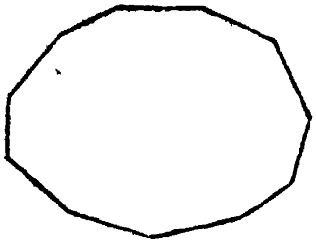
Draw a decagon.



DEF. 49. A figure with eleven sides is called an undecagon.

Write the word UNDECAGON.

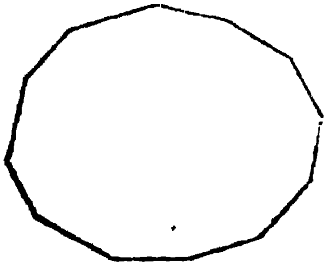
Draw an undecagon.



DEF. 50. A figure with twelve sides is called a dodecagon.

Write the word DODECAGON.

Draw a dodecagon.



DEF. 51. Polygons, that have unequal sides and angles, are called irregular polygons.

Write the words IRREGULAR POLYGONS.

The polygons, which you have just drawn, were therefore all irregular polygons.

DEF. 52. Polygons that have all their sides and angles equal are called regular polygons.

Write the words REGULAR POLYGONS.

The above definitions apply to plane figures bounded by right lines.

Plane figures may also be bounded by curves, as was before explained.

DEF. 53. A circle is a plane figure bounded by a curved line, every part of which curve is equally distant from a point within it.

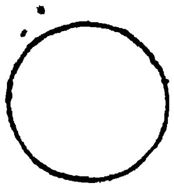
Write the word CIRCLE.

DEF. 54. The point within the circle, from whence all parts of the curve are equally distant, is called the center.

Write the words CENTER OF A CIRCLE.

You will now draw a circle.

Open your compasses to any width, keep one leg fixed, and with the other leg make a sweep all round; when you do this you describe a circle. The point, where the fixed leg stood, is the center, as was just explained.



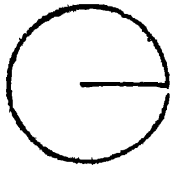
DEF. 55. The sweep made with the moving leg, that is to say the curve which bounds your figure, is called the circumference of the circle.

Write the words CIRCUMFERENCE OF A CIRCLE.

DEF. 56. The radius of a circle is a right line, drawn from the center to the circumference.

Write the word RADIUS.

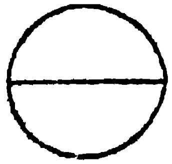
Draw a radius.



DEF. 57. The diameter of a circle is a right line drawn through the center, as far as the circumference, on both sides.

Write the word DIAMETER.

Draw a diameter.



The diameter of a circle is, as you may perceive, exactly double the radius.

DEF. 58. An arc of a circle is any part of the circumference.

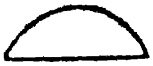
Write THE ARC OF A CIRCLE.

Rub out your diameter and part of the circumference, and an arc will remain; the nature of which, as you may remember, was explained before.

DEF. 59. A chord is a right line joining the extremities of an arc.

Write the words CHORD OF AN ARC.

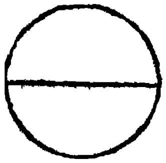
Draw the chord of your arc.



DEF. 60. A segment of a circle is any part of it, which is bounded by a chord and an arc; therefore the figure you have just drawn is a segment.

Write the SEGMENT OF A CIRCLE.

Rub out your figure: describe a new circle, and draw a diameter to it.



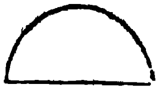
DEF. 61. The diameter divides the circle into two segments, which are equal to each other, and are called semicircles or half circles.

Write the word SEMICIRCLE.

DEF. 62. The half circumference of a circle is called the semi-circumference.

Write the word SEMI-CIRCUMFERENCE.

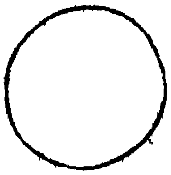
Rub out half your circle.



The figure now remaining is a semicircle: it is bounded by the diameter and by the semi-circumference.

Rub out your figure.

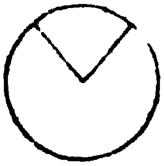
Describe a new circle.



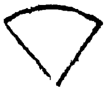
DEF. 63. Any part of a circle, which is bounded by two radii, and an arc, is called a sector.

Write the word SECTOR.

Draw two radii to your circle, forming any angle you please.



Rub out all the rest of the circumference of the circle, except the arc, which is bounded by your two radii, and the figure remaining will be a sector.

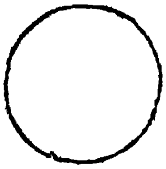


Rub out your sector.

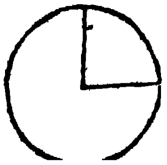
DEF. 64. A quadrant or quarter of a circle is a sector which has a quarter of the circumference for its arc; or whose two radii are perpendicular to each other.

Write the words QUADRANT OR QUARTER OF A CIRCLE.

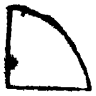
Describe a new circle.



Draw two radii perpendicular to each other.



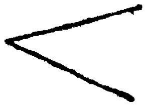
Rub out all the circumference, except the arc contained between your two radii; and the figure that remains will be a quadrant.



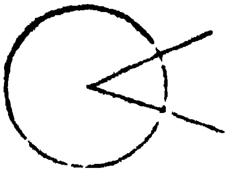
I will now give you a full explanation of the nature of angles, which could not have been well understood, if it had been introduced before the properties of the circle were defined.

If a circle is described, with any radius, from the angular point of a right-lined angle, as a center; the angle will be measured by the arc, which is bounded by the two right lines that form the angle.

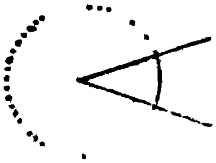
Draw a right-lined angle.



From the angular point, as a center, describe a circle with any radius.



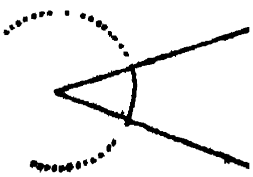
Dot every part of the circumference except the arc that is comprehended between the two right lines, which form the angle.



The proportion which this arc bears to the whole circumference, determines the magnitude of the angle.

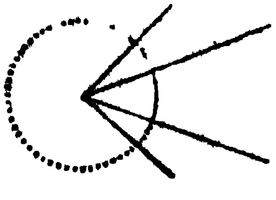
On examining this figure with attention, you will understand clearly the nature of an angle as it was formerly defined.

Produce your lines to a much greater length.



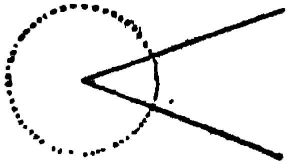
You see that this makes no difference in the proportional size of the arc which measures your angle.

From the center of the same circle draw two shorter lines, on contrary sides of your two former lines.

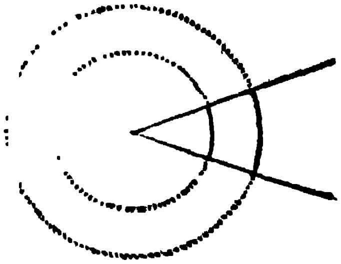


You now see, that the arc, which is bounded by these two short lines, is much larger, and consequently bears a much greater proportion to the whole circumference, than the former arc.

This shows that the magnitude of an angle does not depend on the length of the two lines that form it, but on their opening, or bevel.



Rub out your two last drawn lines, and your original angle will remain.



From the same center, with a greater radius, draw a new circle; and dot all the circumference of it except the arc which is contained between the two lines that form the angle.

This figure now shows that the length of the radius, with which you describe the arc, that is to measure an angle, makes no difference; because if you compare these two arcs together, you will find, that the small arc bears the same proportion to the circumference of the small circle, as the large arc bears to the circumference of the large circle.

For instance, if you take the length of the small arc, in your compasses, and apply it to the circumference of the small circle, and it should take eight times in going round; you will find that on applying the length of the large arc to the circumference of the large circle, it will also take exactly eight times to go round; and the same will happen in any other proportion greater or less than one-eighth.

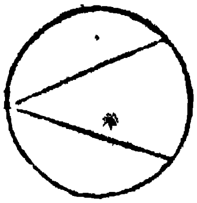
The Teacher will explain this practically on the board.

The usual mode of measuring angles, is by degrees, or equal parts, into which the circumference of a circle is supposed to be divided. This will afterwards be explained to you when you are further advanced.

DEF. 65. An angle at the circumference of a circle, is one whose angular point is any where in the circumference.

Write AN ANGLE AT THE CIRCUMFERENCE.

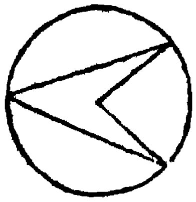
Describe a circle and draw an angle in it, whose angular point is at the circumference.



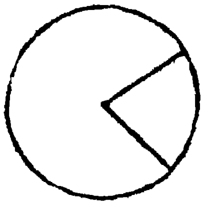
DEF. 66. An angle at the center of a circle is one whose angular point is at the center.

Write AN ANGLE AT THE CENTER.

Draw an angle at the center, standing on the same arc as the former angle.



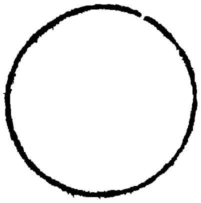
Rub out your angle at the circumference, and the angle at the center only remains.



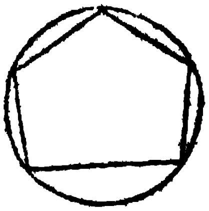
DEF. 67. A right-lined figure is said to be inscribed in a circle, when all the angular points are in the circumference of the circle.

Write A FIGURE INSCRIBED IN A CIRCLE.

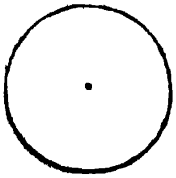
Rub out the angle at the center of your circle.



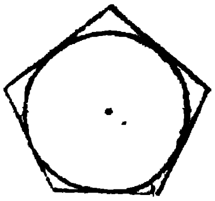
Inscribe a pentagon in your circle.



DEF. 68. A circle is said to be inscribed in a figure, when all the sides of the figure are tangents to the circumference.



Draw a circle.

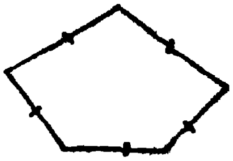


Draw five lines touching, but not cutting, the circumference in different points; and produce these tangents till they meet and form a pentagon.

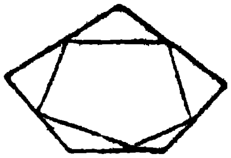
The circle, which you first drew, is inscribed in the pentagon.

DEF. 69. One right-lined figure is said to be inscribed in another, when all the angular points of the one figure are in the sides of the other.*

Draw a pentagon.



Mark a point any where in each side of it.



Join the five points thus marked by as many right lines, which will give you a new pentagon.

The last drawn pentagon is inscribed in the former.

DEF. 70. A line that cuts a circle is called a secant.

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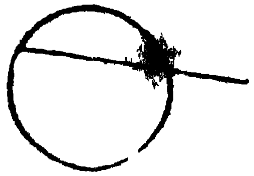
* *DEF.* When one figure is inscribed in another, the latter figure is also said to circumscribe the former.

This definition, which appeared in the original manuscript of this work, has been omitted; because it was found by experience, that almost all the learners were apt to confound the terms "inscribe" and "circumscribe."

Write the word SECANT.

Describe a circle.

Mark any point in the circumference.



And from this point draw a line across the circle, cutting the circumference in some other point.

The line which you have just drawn is a secant.

In the directions for drawing the above, there are three distinct orders or words of command, although only one figure has been made to illustrate the whole of them. The same circumstance will be observed in several other parts of this course, where instead of a distinct figure for every distinct order, one figure only will be given for two or three orders; whenever the nature of the operation to be performed, after such orders, is so exceedingly simple, as to afford no room for mistake, on the part of the Teacher, for whose guidance the figures herein contained are principally intended.

The Teacher is recommended, however, in almost all these cases, to see and examine the figures of the learners, after every individual order; observing always the general rule, of showing them the example of every thing that is to be done, himself; by previously drawing, upon the board, such part of the figure contained in this book, as each successive step of the operation requires, in conformity to its respective word of command.

Rub out your figures.

I will now explain to you the nature of some of the principal solids.

The Teacher must have wooden solids of the proper form, in order to illustrate the following definitions.

The base of a solid is the undermost side of it.

The vertex of a solid is the highest point, or peak of it.

The altitude of a solid is the perpendicular height of it, measuring from the vertex to the base.

Write the words BASE OF A SOLID.
VERTEX OF A SOLID.
ALTITUDE OF A SOLID.

All these terms, as you will recollect, were before applied in the same sense to plane figures or superficies.

A solid is often called a body, to distinguish it from a superficies, which as was before explained has no substance but merely shape or figure.

DEF. 71. A prism is a solid whose two ends are plane figures, equal, exactly alike, and parallel to each other. The sides of a prism must be parallelograms.

A prism takes its name from the figure of its ends, which may for instance be triangles, quadrilaterals, pentagons, &c.

This which I now show you is a triangular prism; observe its two ends they are triangles, exactly alike, and placed parallel to each other.

All its sides you may also perceive are parallelograms.

Write the words TRIANGULAR PRISM.

I will now show you a quadrilateral prism.

Observe its two ends, they are quadrilateral figures, exactly alike, and placed parallel to each other.

Its sides are also parallelograms.

Write the words QUADRILATERAL PRISM.

I will now show you a pentagonal prism.

Its ends as you observe are pentagons, exactly alike, and are placed parallel to each other. And its sides are all parallelograms.

Write the words PENTAGONAL PRISM.

In like manner, there may be hexagonal prisms, heptagonal prisms, &c.

An upright or rectangular prism, is that which has all its sides perpendicular to the base.

An oblique prism is that whose sides are not perpendicular to the base.

Write the words UPRIGHT OR RECTANGULAR PRISM:
OBLIQUE PRISM.

Observe these two prisms: they are both triangular prisms: this is an upright prism: the other is an oblique prism.

DEF. 72. A parallelopiped is a solid body with six faces or sides, all of which are parallelograms; and every two opposite sides of which are equal, exactly alike, and parallel to each other.

Write the word PARALLELOPIPED.

A rectangular parallelopiped is that whose six sides are all rectangles.

Write the words RECTANGULAR PARALLELOPIPED.

A brick for instance is a rectangular parallelopiped.

An oblique parallelopiped is that whose six sides are not rectangles.

Observe these two parallelopipeds: one of them is a rectangular parallelopiped; the other is an oblique parallelopiped.

DEF. 73. A cube is a solid, whose length, breadth, and thickness, are equal: it has six faces or sides, each of which is a square, and all these faces are equal.

A pair of dice for instance are cubes.

Write the word CUBE.

DEF. 74. A cylinder is a regular solid whose two ends are circles, and the sides of which are perpendicular to the ends.

A rolling-stone for instance is a cylinder.

Write the word CYLINDER.

DEF. 75. The axis of a cylinder is a right line, supposed to pass through the centers of the two circles which form the ends of it.

Write AXIS OF A CYLINDER.

In a rolling-stone or in any cylinder which you wish to move about, the pivots, by which you move it, must pass through the axis, or it will not move regularly.

When a cylinder or any other solid body moves round upon an axis, like a wheel upon its axle-tree, it is said to revolve.

Write A SOLID BODY MAY REVOLVE.

When a solid body moves round or revolves, one turn round is called a revolution.

Write REVOLUTION OF A SOLID BODY.

DEF. 76. A pyramid is a solid which has any right-lined plane figure for its base, and whose sides are triangles, that terminate in the same point or vertex above the base.

Write PYRAMID.

A pyramid is called after its base like a prism: the pyramid which I now show you, is for instance a triangular pyramid.

Write TRIANGULAR PYRAMID.

I will now show you a square pyramid.

The pyramids of Egypt, which you may have heard of, are square pyramids.

Write SQUARE PYRAMID.

I will next show you a pentagonal pyramid.

Write PENTAGONAL PYRAMID.

Thus there may be hexagonal pyramids, heptagonal pyramids, &c.

DEF. 77. A cone is a round pyramid, being exactly the same kind of solid, only that it has a circle for its base.

Write the word CONE.

A sugar-loaf for instance is a cone.

A tile-kiln is also shaped like a cone.

The axis of a cone is a right line supposed to be drawn from its vertex to the center of the circle which forms the base.

Write AXIS OF A CONE.

DEF. 78. A globe or sphere is a regular solid body perfectly round, every part of the surface of which is equally distant from a point within it, which is its center.

Write the words GLOBE OR SPHERE.

A cannon-ball for instance is a globe or sphere.

DEF. 79. The diameter of a globe or sphere is any right line supposed to pass through the center of it, from one side to the other.

Write the words DIAMETER OF A GLOBE OR SPHERE.

DEF. 80. The axis of a globe or sphere is the diameter or right line passing through the center, about which the globe or sphere is supposed to revolve.

Write THE AXIS OF A GLOBE OR SPHERE.

A globe or sphere may have many diameters, but it has only one axis.

DEF. 81. Any line or surface which is quite level is said to be horizontal.

Write a HORIZONTAL LINE.

Write also a HORIZONTAL PLANE OR SUPERFICIES.

The surface of water in a lake for instance is horizontal.

DEF. 82. Any line or plane, which is perpendicular to a horizontal surface, is called a vertical line or plane.

Write VERTICAL LINE.

Write also VERTICAL PLANE OR SUPERFICIES.

If you hang a plummet by a string, as soon as it shall leave off swinging, the string will form a vertical line.

In short a vertical line and a plum line are the same thing.

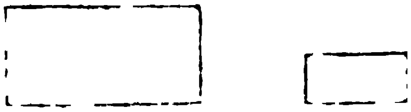
The side of a house, being built according to the plum line, is a vertical plane.

DEF. 83. Figures or solids which are exactly like each other in form, but differ in size, are said to be similar.

Write SIMILAR FIGURES OR SOLIDS.

Similar figures have the same number of sides, and all the corresponding sides of the one figure are proportional to those of the other.

Here the Teacher will draw two rectangles upon the board, making the base of the one double that of the other, and making the first also double in height.



The two rectangles which I have just drawn are similar figures; for every side of the large one is exactly double of the corresponding side of the other.

By corresponding sides, I mean those sides which have the same position. For instance, the base of the one rectangle corresponds to the base of the other. The top of the one, in like manner, corresponds to the top of the other. The left and right sides of each are also corresponding sides.

I will now show you two similar solids.

Here the Teacher will produce two rectangular parallelopipeds; the length, breadth, and thickness of one of which must be exactly double of the same dimensions in the other.

The height of the large solid is double that of the small one. Its length and breadth are also in the same proportion.

They have each the same number of faces or sides, and their corresponding faces are all similar rectangles.

The above definitions are as many as appear necessary at present. We shall now proceed with some more problems, in addition to those which it was thought convenient to intermix with the definitions.

PROBLEM X.

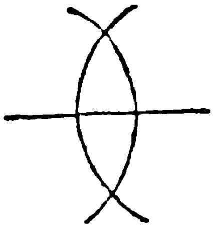
TO BISECT A GIVEN LINE.

To bisect any thing is to divide it into two equal parts.

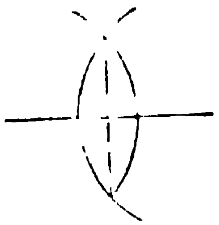
Write BISECT OR DIVIDE INTO TWO EQUAL PARTS.

Draw a right line to represent the given line.

From the two ends of it, as centers, describe arcs intersecting each other both above and below the line.



Join the points of intersection of these arcs by a right line.



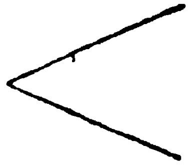
This right line will bisect your given right line, or divide it into two equal parts, as was required.

Rub out superfluous arcs, and your problem is performed.

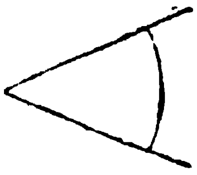
PROBLEM XI.

TO BISECT A GIVEN ANGLE.

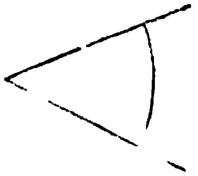
Draw two right lines forming any angle you please, to represent the given angle.



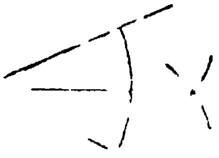
From the angular point, as a center, with any convenient radius, describe an arc intersecting both your lines.



From the two points of intersection, as centers, with any convenient radius, describe arcs intersecting each other, opposite to the angular point.



Draw a right line from the angular point, to the point of intersection of the two last described arcs.



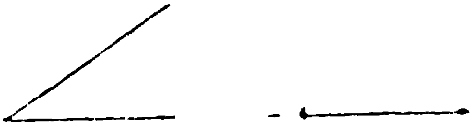
This right line will bisect your given angle, or divide it into two equal parts: your problem is therefore performed.

PROBLEM XII.

AT A GIVEN POINT, IN A GIVEN RIGHT LINE, TO MAKE AN ANGLE EQUAL TO A GIVEN ANGLE.

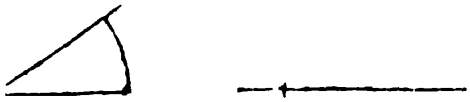
Draw an angle to represent your given angle.

Draw also a right line to represent your given right line, and mark a point, in any part of it, to represent your given point.

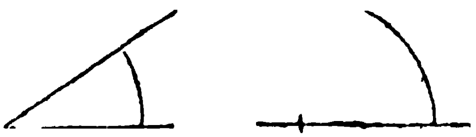


It was before explained that angles are always measured by the arc of a circle.

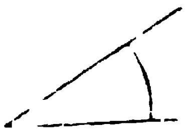
From the angular point of your given angle, with any radius, describe an arc to measure it.

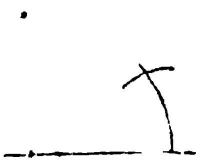


From the given point in the given line, as a center, describe an arc on one side of the line with the same radius.

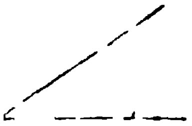


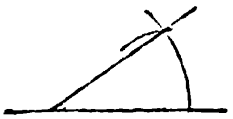
Take the length of your first arc in your compasses, and from the point where the second arc meets the given line, as a center, describe a third arc intersecting the second.





Draw a right line from the given point, through the point of intersection of these two arcs; and your problem will be performed.



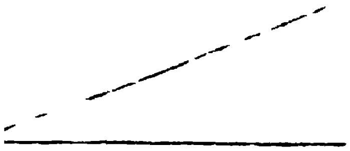


PROBLEM XIII.

TO DIVIDE A GIVEN LINE INTO ANY NUMBER OF EQUAL PARTS.

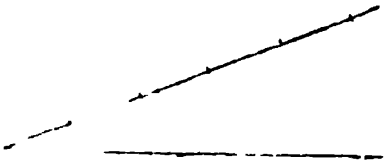
Draw a right line to represent your given line. We shall suppose that it is to be divided into five equal parts.

Draw another line forming an angle with it: an acute angle is best.

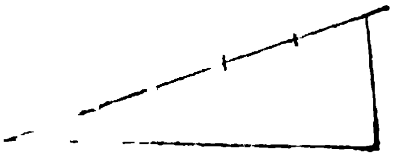


Take any opening in your compasses, such as you imagine will nearly be equal to one of the proposed divisions, and from the angular point, set off this distance upon the second line, as many times as there are to be equal parts in the given right line.

In the present instance we must mark it five times.

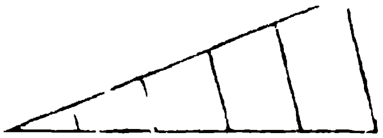


From the last of these points, draw a right line to the extremity of your given right line; this will form a triangle.



Through all the former points draw right lines parallel to the last drawn line.

The points where these parallels meet the given right line, will divide it into the required number of equal parts.



Rub out superfluous lines; and your problem is performed.

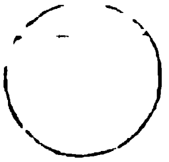
This is a problem proper for you to know: but you must take notice, that unless it is very accurately performed, it is always better to divide any line into equal parts by your compasses, making a number of trials till you get the exact distance.

PROBLEM XIV.

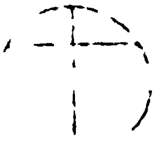
TO FIND THE CENTER OF A GIVEN CIRCLE.

Draw a circle to represent your given circle, the center of which we shall suppose to be lost.

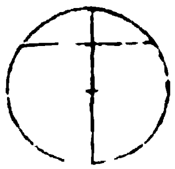
Draw also any chord.



Bisect your chord, and draw a perpendicular through the middle of it, which you will produce as far as the circumference on both sides.



Bisect the last drawn perpendicular, and mark the middle of it by a point.



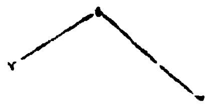
The point last marked is the center of the circle, which you were required to find: your problem is therefore performed.

PROBLEM XV.

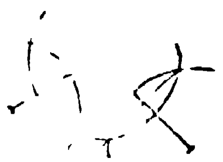
TO DESCRIBE A CIRCLE, THE CIRCUMFERENCE OF WHICH SHALL PASS THROUGH THREE GIVEN POINTS.

Mark three points to represent the given points.

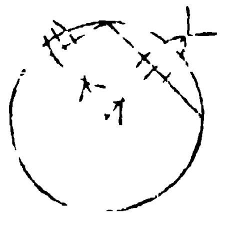
Connect them by two right lines.



Bisect these right lines by perpendiculars, which must be produced till they meet and form an angle.



From the angular point as a center, with the distance of any of the given points as a radius, describe a circle; and the circumference of it will pass through all the three given points . consequently your problem is performed.



PROBLEM XVI.

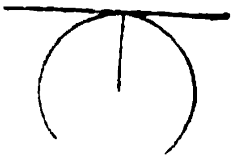
THROUGH A GIVEN POINT TO DRAW A TANGENT TO A GIVEN CIRCLE.

METHOD 1. We shall first suppose the given point to be in the circumference of the circle.

Draw a circle to represent the given circle, and mark a point in the circumference of it to represent the given point.

From this point draw a radius.

Through the given point, in the circumference of the circle, draw a right line perpendicular to this radius.

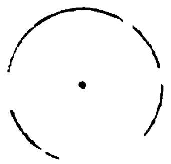


The last drawn line is the tangent required: your problem is therefore performed.

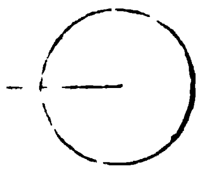
Rub out radius and tangent, leaving only your original circle.

METHOD 2. We shall now suppose the given point, from whence the tangent must be drawn, to be without the circumference of the circle.

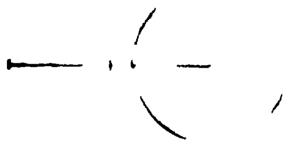
Mark a point without your circle to represent the given point.



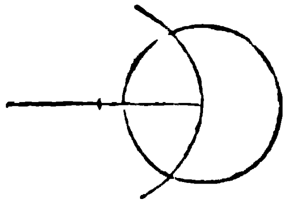
From this point draw a right line to the center of your given circle



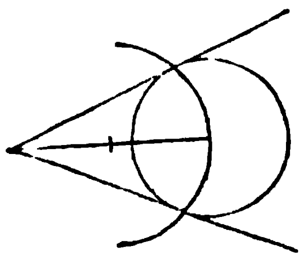
Bisect this right line.



From the middle of it, as a center, with a radius equal to half the line, describe an arc intersecting the circumference of the circle in two places.

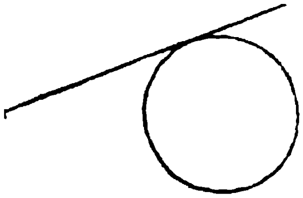


From the given point, draw right lines through these points of intersection.



Both of these lines are tangents to your given circle, and they are drawn from the given point.

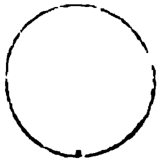
Rub out superfluous lines and arcs, as also one of your tangents; and your problem is performed.



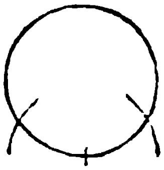
PROBLEM XVII.

IN A GIVEN CIRCLE TO INSCRIBE AN EQUILATERAL TRIANGLE.

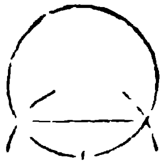
Draw a circle to represent the given circle, and mark a point in the circumference of it.



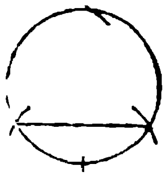
From this point as a center, with the radius of your circle, make two intersections on the circumference.



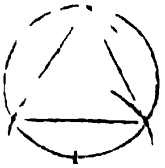
Join the two last points by a right line, which will be one side of the required triangle.



Take the length of this line in your compasses, and from one of the ends of the line, as a center, describe an arc intersecting the circumference in a new point.



From this last point draw two right lines to the extremities of the first line, which will form a triangle.



This triangle is an equilateral triangle, and it is inscribed in the given circle: your problem is therefore executed.

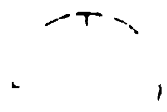
PROBLEM XVIII.

IN A GIVEN CIRCLE TO INSCRIBE A SQUARE.

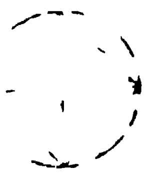
Draw a circle to represent your given circle.

Draw a diameter.

Draw another diameter perpendicular to the former.



Connect the extremities of your diameters by four right lines.



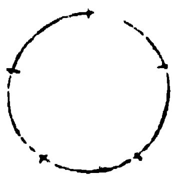
Rub out your two diameters; and the four remaining right lines will form a square inscribed in the given circle: your problem is therefore executed.

PROBLEM XIX.

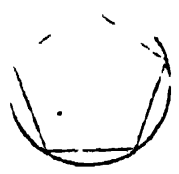
IN A GIVEN CIRCLE TO INSCRIBE A REGULAR PENTAGON.

Draw a circle to represent the given circle.

Divide the circumference of it into five equal parts, by trials with your compasses.



Join the points marked on the circumference by five right lines.



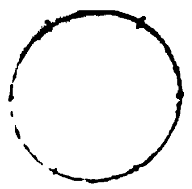
The above five lines form a regular pentagon. Your problem is therefore performed.

PROBLEM XX.

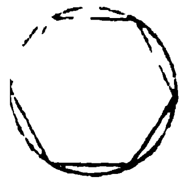
IN A GIVEN CIRCLE TO INSCRIBE A REGULAR HEXAGON.

Draw a circle to represent the given circle.

Take the radius of this circle in your com and apply it round the circumference as often as it will go. This will be exactly six times.



Join the six points upon your circumference by six right lines; and your problem is performed.

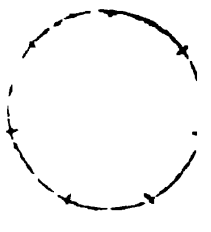


PROBLEM XXI.

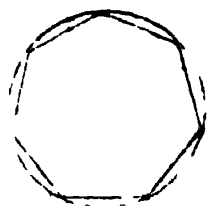
IN A GIVEN CIRCLE TO INSCRIBE A REGULAR HEPTAGON.

Draw a circle to represent your given circle.

Divide the circumference of it into seven equal parts, by trials with your compasses.



Join the points upon your circumference by seven right lines; and the problem is performed.

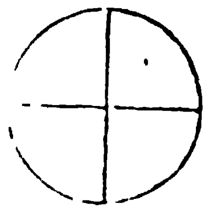


PROBLEM XXII.

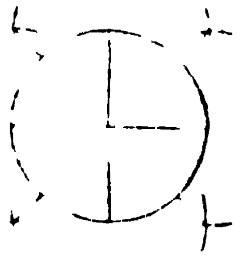
IN A GIVEN CIRCLE TO INSCRIBE A REGULAR OCTAGON.

Draw a circle to represent the given circle.

Draw two diameters perpendicular to each other, which will divide the circumference into four equal parts.

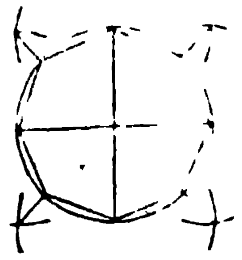


Bisect each of these four equal parts.



The circumference of your given circle is now subdivided into eight equal parts, which are marked by points.

You will join the above points by as many right lines.

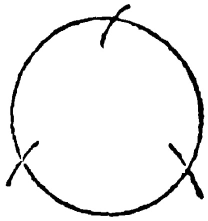


The eight right lines, which you have just drawn, form a regular octagon: your problem is therefore performed.

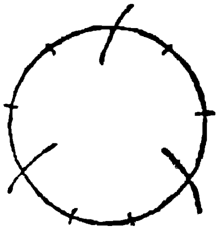
PROBLEM XXIII.

IN A GIVEN CIRCLE TO INSCRIBE A REGULAR NONAGON.

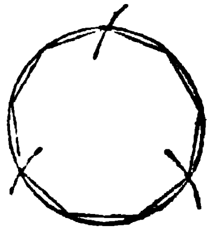
Draw a circle to represent the given circle; and divide the circumference of it into three equal parts.



The length of one of these parts is found in the way explained in a former problem, by applying the radius twice to any part of the curve.



Subdivide each of these parts also into three equal parts, by trials with your compasses.

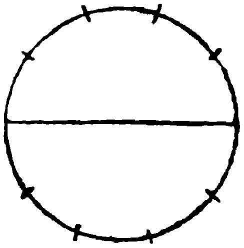


Join the nine points marked on the circumference of your circle by as many right lines, which will form a regular nonagon: consequently the problem is executed.

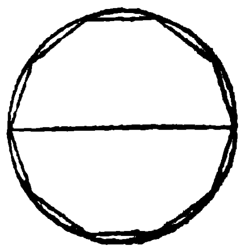
PROBLEM XXIV.

IN A GIVEN CIRCLE TO INSCRIBE A REGULAR DECAGON.

Draw a circle to represent the given circle; and draw a diameter, which will divide it into two semicircles.



Divide each of your two semicircles into five equal parts.



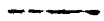
Join the ten points now marked on the circumference by ten right lines; which will form a regular decagon: your problem is therefore performed.

In like manner a regular undecagon, dodecagon, or any other kind of polygon, may be inscribed in a given circle; by previously dividing the circumference into as many equal parts as the required figure has sides.

PROBLEM XXV.

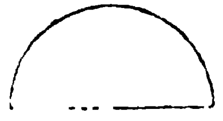
TO DESCRIBE A REGULAR POLYGON, WHOSE SIDES SHALL EACH BE EQUAL TO A GIVEN RIGHT LINE.

Draw a right line to represent the given right line, and also the side of the proposed polygon.



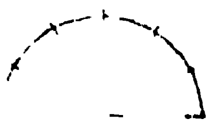
Let us suppose that the polygon required to be made is a regular hexagon.

From the left extremity of the given line, as a center, with the length of it as a radius, describe a semicircle; complete the diameter of this semicircle by producing the line; and dot the produced part.

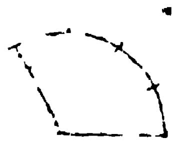


Divide the semicircle into as many equal parts as the proposed polygon has sides.

Your proposed polygon in the present instance being a hexagon, six equal parts will be required.

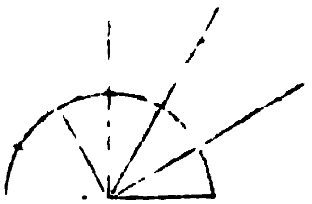


From the left extremity of your given line, which corresponds with the center of your semicircle, draw a right line to the second point, which is marked on the left side of the semicircle.

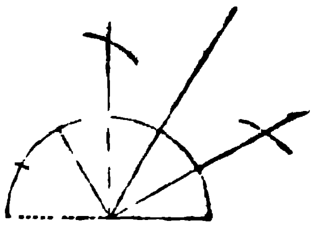


The last drawn line, and the given right line, will form two of the sides of your proposed polygon.

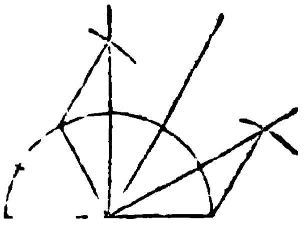
From the center of your semicircle draw right lines cutting the curve, through all the points that are comprehended within the angle, which is formed by these two sides.



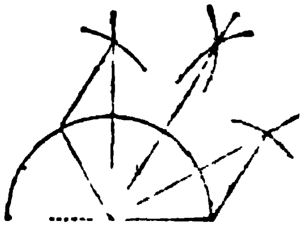
From the outward extremities of the two finished sides of your polygon, with a radius equal to one of the sides, describe arcs intersecting the two nearest right lines which cut the curve.



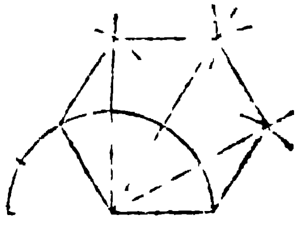
From the center by which these arcs were described, draw right lines to the two points of intersection thus found: and these lines will give you two new sides of your proposed polygon.



From the outward extremities of the two last finished sides of your polygon, with a radius equal to one side, describe arcs intersecting the nearest remaining line or lines which cut the curve.



From the center by which these arcs were described, draw right lines to the points of intersection found; which will give you two new sides.



In the hexagon this will complete your figure.

Rub out your superfluous semicircle, lines, and arcs; and the problem is performed.

Rub out your figure.

Here the Teacher will exercise the learners in performing this problem several times, choosing other polygons, such as a pentagon, a heptagon, an octagon, &c. observing to make such variations, in giving his directions, as the nature of the polygon may require.

In the pentagon, the figure will be completed, before he has given the whole of the above orders which are necessary for a hexagon.

If, on the contrary, the proposed figure is a heptagon, an octagon, or other polygon of more sides than six; the figure will not be completely closed in, after the whole of the above orders have been issued.

In this case, the Teacher will again repeat the two following orders, being the same which were used at the end of the construction of the hexagon.

From the outward extremities of the two last finished sides of your polygon, with a radius equal to one side, describe arcs intersecting the nearest remaining line or lines, which cut the curve.

From the centers, by which these arcs were described, draw right lines to the points of intersection thus found; which will give you two new sides.

These two orders, being repeated as often as proves necessary, will serve for the largest polygons.

There is a more correct method of performing this problem, by drawing a circle, whose radius shall have a certain proportion to the given line, according to the nature of the required polygon; and afterwards inscribing the polygon in this circle.

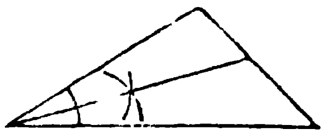
But the proportion which the side of any polygon bears to a circle, in which it is inscribed, is more properly to be found by mensuration, than by practical geometry.

PROBLEM XXVI.

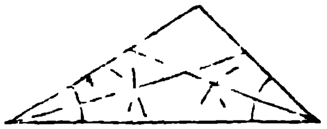
IN A GIVEN TRIANGLE TO INSCRIBE A CIRCLE.

Draw a triangle to represent your given triangle.

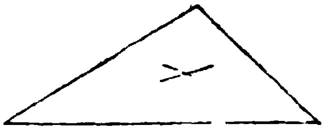
Bisect any angle of it, for instance the angle at the left extremity of the base, by a right line.



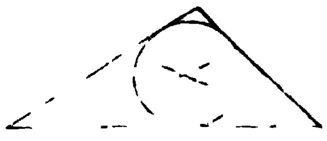
In like manner bisect by a right line any other angle of it, for instance the angle at the right extremity of the base.



Rub out superfluous arcs, and every part of the two bisecting right lines, except the point where they intersect each other. The above point will be the center of your proposed circle.



From this point, drop a perpendicular to any of the sides of your given triangle, for instance to the base; and with this perpendicular, as a radius, describe a circle.



Your problem is performed: you have inscribed a circle in the given triangle.

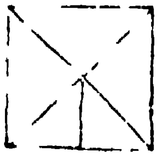
PROBLEM XXVII.

IN A GIVEN SQUARE TO INSCRIBE A CIRCLE.

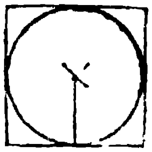
Draw a square to represent the given square.

Draw two diagonals intersecting each other, in a point, which will be the central point of the square.

From this point drop a perpendicular to any side of your square.



Rub out your diagonals; and from the central point of your square, with the perpendicular just drawn, as a radius, describe a circle.



Your problem is performed.

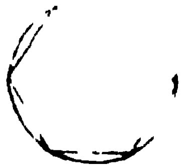
PROBLEM XXVIII.

IN A GIVEN REGULAR POLYGON TO INSCRIBE A CIRCLE.

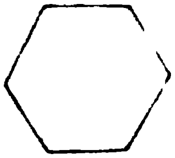
A regular polygon must first be drawn to represent the given polygon. The best way of doing this is by first drawing a circle.

Let us suppose, for instance, that the given polygon is a hexagon.

Draw a circle; and inscribe a regular hexagon in it.



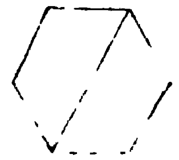
Rub out your circle, and the regular hexagon only remains.



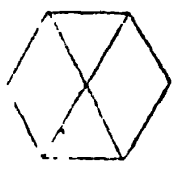
This hexagon represents the given polygon in which you are required to inscribe a circle.

Bisect any angle of your given regular polygon by a right line.

If the polygon has an even number of sides, a diagonal drawn from any angle, to the angle which is directly opposite to it, will bisect either of these angles, at once, without the trouble of measuring them by an arc, &c.

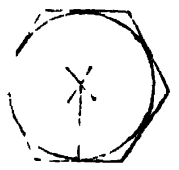


The hexagon having an even number of sides, you may therefore bisect any angle of it, in this manner. Draw a diagonal accordingly.



Bisect in like manner any other angle of your given polygon (no matter which) by a second diagonal intersecting the former. The point where they intersect each other will be the central point of the figure.

Rub out every part of the two bisecting lines except the above point of intersection; and from this point, drop a perpendicular to any side of your given polygon.



From the central point of your given polygon, as a center, with the perpendicular as a radius, describe a circle.

This circle is inscribed in your given polygon: the problem is therefore performed.

This rule applies to all other kinds of regular polygons, as well as hexagons; and is in fact the same method as that which was used in the two last problems, in respect to the triangle and the square.

Here the Teacher may cause the learners to inscribe a circle in a regular pentagon, heptagon, &c. if he thinks proper, making such variations in his orders as the nature of the new polygon may require.

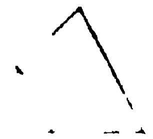
It is impossible to inscribe a circle properly in an irregular polygon; because all the sides of an irregular figure cannot be made to touch a circle.

PROBLEM XXIX.

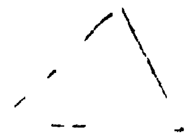
ABOUT A GIVEN TRIANGLE TO DESCRIBE A CIRCLE.

Draw a triangle to represent your given triangle.

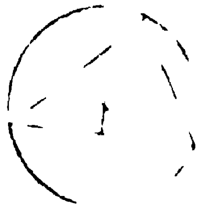
Bisect any two sides of it, for instance the base and the left leg.



From the middle points of the two bisected sides, draw perpendiculars which you must produce until they intersect each other, either within or without the triangle.



From the point of intersection of these two perpendiculars, as a center, with the distance between this point and any of the angular points of your figure, as a radius, describe a circle.



Your problem is now performed it is similar to a former problem, wherein you were taught to describe a circle through three given points.

PROBLEM XXX.

ABOUT A GIVEN EQUILATERAL TRIANGLE, SQUARE, OR REGULAR POLYGON, TO DESCRIBE A CIRCLE.

Draw { an equilateral triangle
a square
a regular polygon } to represent the given figure.

If the Teacher should think proper to direct the learners to draw a regular polygon, he must of course specify what kind of polygon he requires.

Find the central point of your given figure, by the method shown in the former problems; that is to say, if it is a square, draw two diagonals intersecting each other: but if it is an equilateral triangle or a regular polygon, bisect any two angles by right lines intersecting each other.

This process is so very simple, that it has not been judged necessary to insert figures for the guidance of the Teacher, who may be able to draw the necessary figures on the board, according to his own judgement, without having any copy before him.

If an equilateral triangle is chosen, he may however, if he thinks proper, refer to the two or three first figures of Problem 26, because the method there used, in finding the central point of the given triangle, will of course also apply to this problem.

If a square is chosen, he may, in like manner, refer to the first figure of Problem 27, which, provided the perpendicular were rubbed out, would serve for this problem.

If a regular polygon (for instance a hexagon) is chosen, he may refer to the four first figures in Problem 28, the method for drawing which will equally apply to the present problem.

Having found the central point of your given figure, rub out all superfluous lines, &c. leaving only the given figure and the said point.

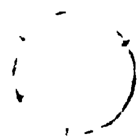
From the central point of your given $\left\{\begin{array}{l}\text{equilateral triangle}\\ \text{square}\\ \text{polygon}\end{array}\right\}$ as a center, with the distance from this to any of the angular points of your figure, as a radius, describe a circle.

The figures to correspond with this order will be exactly or nearly the same, as the last figures in Problems 17, 18, 19, 20, 21, 22, 23 and 24, to which the Teacher may refer, if he thinks proper; but this can hardly be necessary, the thing being so very simple.

PROBLEM XXXI.

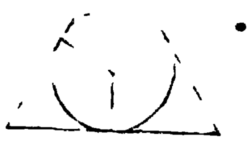
ABOUT A GIVEN CIRCLE TO DESCRIBE AN EQUILATERAL TRIANGLE.

Draw a circle to represent your given circle, and divide the circumference of it into three equal parts.

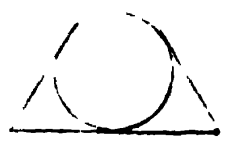


Through the points marked on the circumference draw three tangents, and produce them till they form a triangle.

The mode of finding tangents, as was formerly explained to you, is first to draw right lines or radii to the center of the circle from the given points, and then through these points to draw perpendiculars to the above radii.



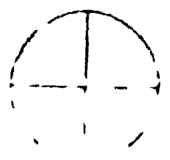
Rub out your radii and the problem is performed; the triangle which you have made being an equilateral triangle, and being described about the given circle, as was required.



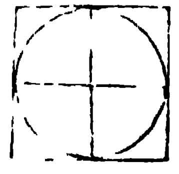
PROBLEM XXXII.

ABOUT A GIVEN CIRCLE TO DESCRIBE A SQUARE.

Draw a circle to represent the given circle; and draw two diameters intersecting each other at right angles.



Through the extremities of these two diameters, draw four tangents to the circumference, which must of course be made perpendicular to the diameters.



Rub out your diameters; the four tangents just drawn will form a square: your problem is therefore executed.

PROBLEM XXXIII.

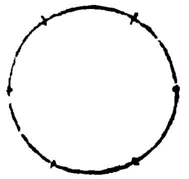
ABOUT A GIVEN CIRCLE TO DESCRIBE A REGULAR POLYGON.

Draw a circle to represent your given circle.

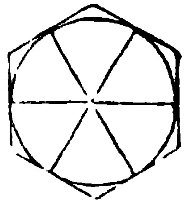
The circumference of the given circle must be divided into as many equal parts as the proposed polygon has sides.

Let us suppose that it is required to describe a regular hexagon about the given circle.

You will in this case divide the circumference of your circle into six equal parts.



Through each point marked on the circumference draw a tangent to the curve, and produce these tangents till they meet and form a polygon.



Rub out the radii by means of which you found your tangents.

Your problem is now performed: the polygon which you have just drawn is a regular polygon of the nature required; and it is described about the given circle.

Here the Teacher may cause the learners to describe some other polygons (not hexagons) about a given circle.

This mode of describing a polygon about a given circle, unless it is executed with the greatest care, is not likely to prove so accurate, as another method which more properly forms a part of mensuration; and depends upon a knowledge of the proportion which the radius of a circle bears to a regular polygon of any kind either inscribed within the circle or described about it.

PROBLEM XXXIV.

TO MAKE A RECTANGLE SIMILAR TO A GIVEN RECTANGLE.

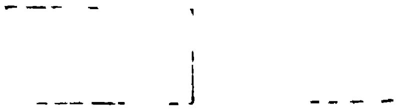
Draw a rectangle to represent the given rectangle.

The rectangle which you are to make, may be required to be drawn, either on a larger or smaller scale, than your given rectangle. The rule for drawing it is the same in both cases.

We shall first draw it on half the scale.

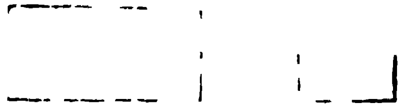
Draw a right line equal to half the base of your given rectangle.

This right line is to be the base of your new rectangle.

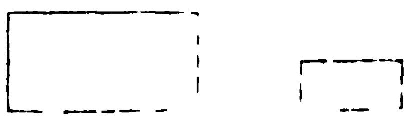


From the extremities of your new base raise perpendiculars, and make each of these perpendiculars half the height of the given rectangle.

These perpendiculars will form the two sides of your new rectangle.



Join the extremities of the above perpendiculars by a right line, which will complete your new rectangle.



Your problem is performed. You have drawn a rectangle similar to a given rectangle on half the scale.

Here the Teacher may cause the learners to draw some more proportional rectangles on different scales, such as two thirds, &c.

PROBLEM XXXV.

TO MAKE A TRIANGLE SIMILAR TO A GIVEN TRIANGLE.

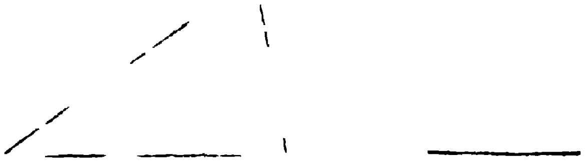
Draw a triangle to represent your given triangle.

Let us suppose that the new triangle, which we are to make, is to be drawn on half the scale.

Half the scale is given here, and in some of the following problems, merely as an example; therefore the Teacher is not bound down to this proportion, but may pitch upon any other he thinks proper, observing only not to make too great an inequality in the scale of the two figures, lest the smaller one should be indistinct.

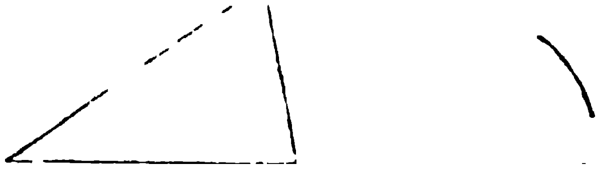
In this, and in all the following problems, which relate to the construction of similar figures, he must cause the learners to draw on as large a scale as their slates will conveniently permit.

Draw a right line equal to half the base of your given triangle.

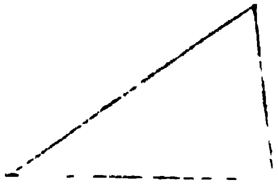
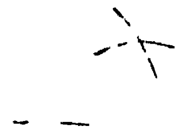


This right line is to be the base of your new triangle.

Take in your compasses a distance equal to one half of the left leg of your given triangle, and with this distance as a radius, from the left extremity of your new base, as a center, describe an arc above it.

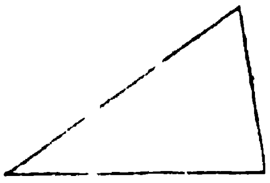
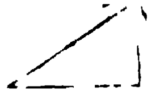


From the other extremity of your new base, as a center, with a radius equal to one half of the right leg of your given triangle, describe another arc intersecting the former.

The point of intersection of these arcs will be the vertex of your new triangle.

Draw right lines from this point to the two extremities of the base, and complete your triangle.

Your problem is now executed. You have drawn a triangle on a smaller scale, similar to your given triangle.

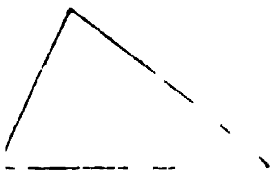
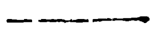
Rub out your figure.

ANOTHER METHOD OF PERFORMING THE SAME PROBLEM.

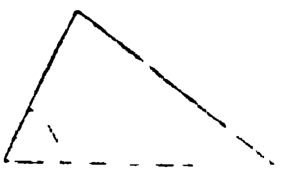
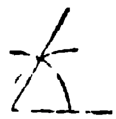
I will now teach you another method of drawing proportional triangles.

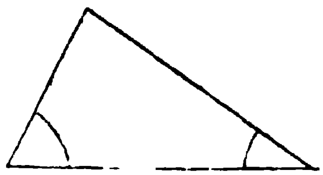
Draw a triangle to represent your given triangle.

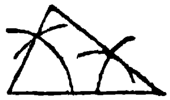
Draw also a right line for the base of your new triangle, equal to one half of the base of your given triangle.

At the left extremity of your new base, make an angle, equal to the angle which is formed by the base and the left leg of your given triangle.





At the other extremity of your new base make an angle, equal to the angle which is formed by the base and the right leg of your given triangle.

The lines which you have just drawn in order to form the above angles will meet each other in a point.

This point will be the vertex of a new triangle of which the above lines form the legs.

This new triangle is similar to your given triangle: your problem is therefore performed.

REMARKS ON PROBLEM XXXV.

Upon this problem the art of Land-Surveying is principally founded.

The land-surveyor measures a long line as a base, on some convenient place, such as a large heath or common. From one extremity of this base he takes an angle to some distant object, for instance a steeple. From the other extremity of his base he also takes an angle to the same object.

This is generally done by means of an instrument called a theodolite.

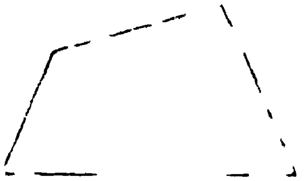
He then lays down the base upon his plan, on a small scale of any number of inches to a mile that he thinks proper. From the two extremities of the right line which represents his base, he draws two other right lines, forming angles equal to those which he before measured on the ground, and he produces these lines till they meet. These two lines and the base will then form a triangle, the vertex of which shows the point upon the plan where the surveyor must mark his steeple: because the small triangle, drawn upon paper, will be proportional to the large triangle, formed upon the ground, between the two ends of the measured base and the steeple.

Thus by measuring only one base, a great number of distant objects may be laid down correctly in the map of a country; for a great many angles may be taken at the same time, from the two ends of the base.

PROBLEM XXXVI.

TO MAKE A TRAPEZIUM SIMILAR TO A GIVEN TRAPEZIUM.

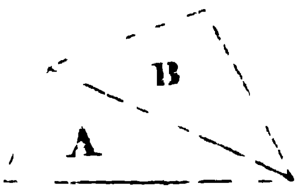
Draw a trapezium to represent your given trapezium.



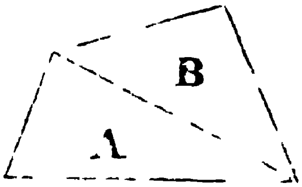
Let us suppose that it is required to make the new trapezium on half the scale of your present figure.

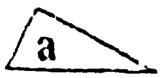
Draw a diagonal which will divide your trapezium into two triangles.

Mark that triangle, which has the same base as the trapezium itself, with a capital A; and mark the other triangle with a capital B.

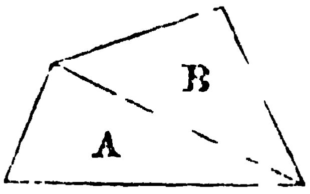


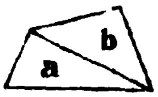
By the rule given in the last problem draw a triangle, similar to the triangle A, on half the scale; and mark this new triangle with a small a.





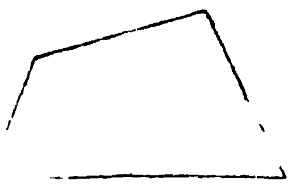
Upon that side, of your last drawn triangle, which corresponds with the diagonal of your given trapezium, draw a new triangle similar to the triangle B, also on half the scale; and mark this new triangle with a small b.

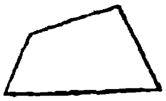




The two triangles, marked with the small letters a, b, form a new trapezium similar to the given trapezium, on half the scale.

Rub out the diagonals, and the superfluous letters; and your problem is performed.



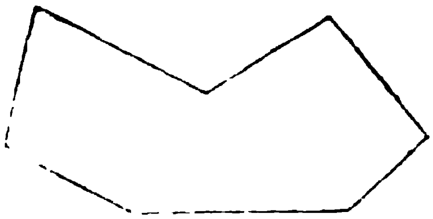


PROBLEM XXXVII.

TO DRAW A FIGURE SIMILAR TO ANY GIVEN RIGHT-LINED IRREGULAR FIGURE.

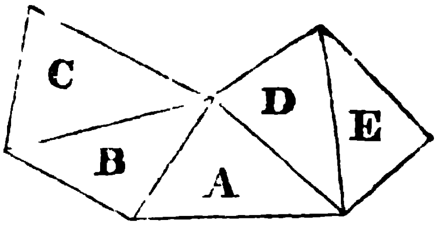
Draw an irregular figure of any number of sides to represent your given figure.

We shall for instance draw a figure of seven sides, or an irregular heptagon.



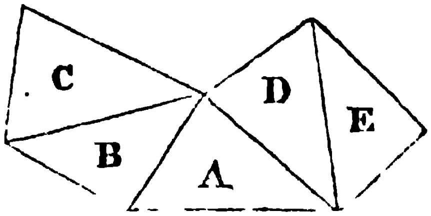
Draw as many diagonals as are necessary to divide it into triangles.

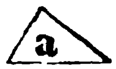
Mark these triangles with the capital letters A, B, C, D, &c.



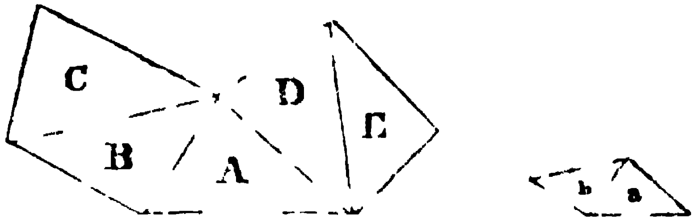
Let us suppose that the proposed similar figure is to be drawn on half the scale of your given figure.

Draw a triangle, on half the scale, similar to that part of your given figure,





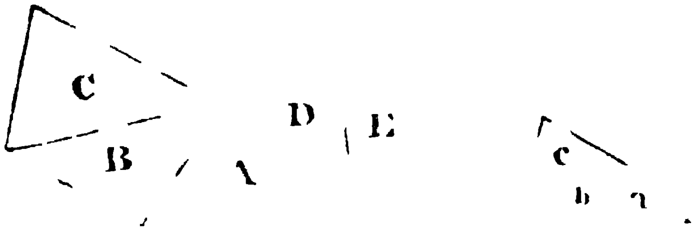
which is marked by the letter A: and mark this new triangle by a small a.



Upon the proper side of the small triangle a, make another triangle, on half the scale, similar to that part of your given figure, which is marked by the letter B

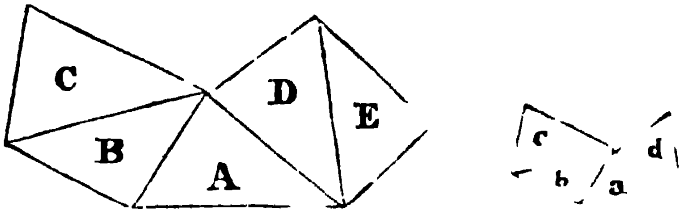
And mark this new triangle by a small b.

The trapezium formed by the two small triangles a and b, is similar to that part of your given figure, which is formed by the two larger triangles A and B.



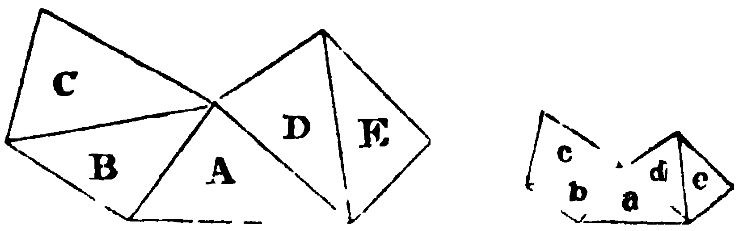
Upon the proper side of the small triangle b, draw another triangle, on half the scale, similar to that part of your given figure, which is marked by the letter C:

And mark this new triangle by a small c



Upon the proper side of the small triangle a, make a triangle, on half the scale, similar to that part of your given figure, which is marked by the letter D:

And mark this new triangle with a small d.

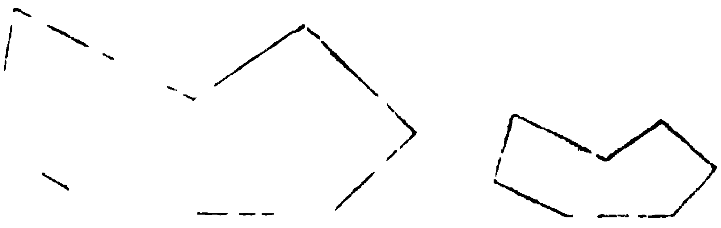


Lastly, on the proper side of your small triangle d, make a triangle, on half the scale, similar to that part of your given figure, which is marked by the letter E:

And mark this last triangle with a small e.

All the triangles in the two figures which are marked with the same letters are similar to each other.

Rub out diagonals and superfluous letters.

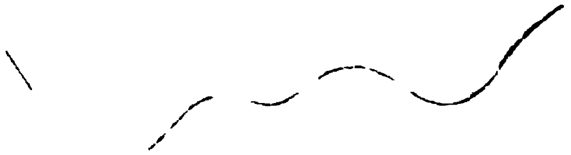


Your problem is performed. You have drawn a figure similar to your given irregular right-lined figure.

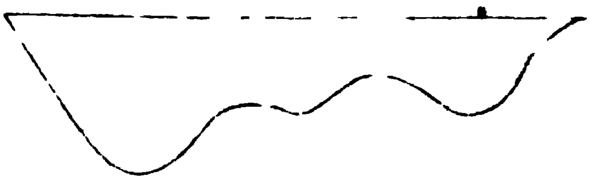
PROBLEM XXXVIII.

TO DRAW A CURVE SIMILAR TO A GIVEN CURVE.

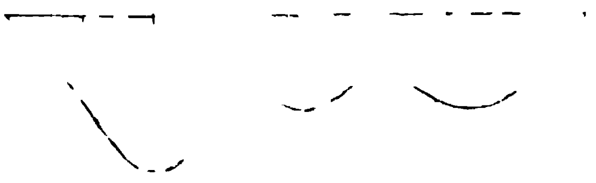
Draw a curve to represent your given curve.



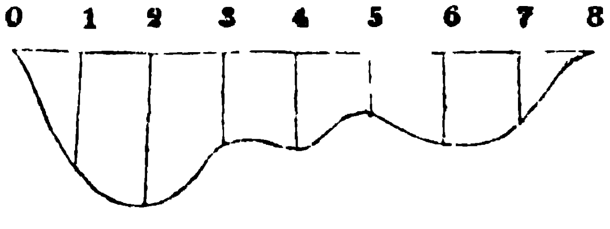
Connect the two extremities of your given curve by a right line.



Divide this right line into any number of equal parts, 8 for instance.



Mark these parts by the numeral figures, 0, 1, 2, 3, 4, 5, 6, 7, 8; and from each point, upon your right line, draw perpendiculars meeting the given curve.

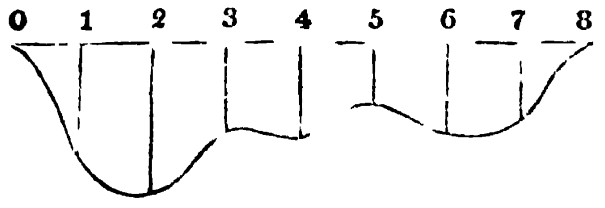


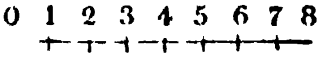
Let us suppose that the proposed curve is to be drawn similar to your given curve, on half the scale.

Draw a right line equal to one half of the first drawn right line in your present figure.

Divide this new line into eight equal parts:

And number each of these parts in the same way as you did those of your former line.



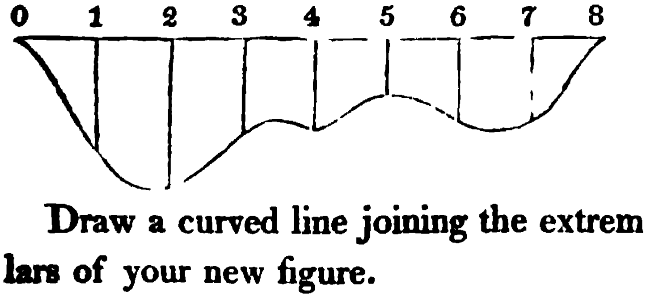


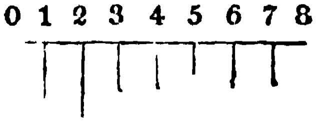
From each point on your new line raise drop perpendiculars.

Let the perpendicular, drawn from the point 1 on your new line, be exactly half of the perpendicular, which is drawn from the point marked 1 in your first figure.

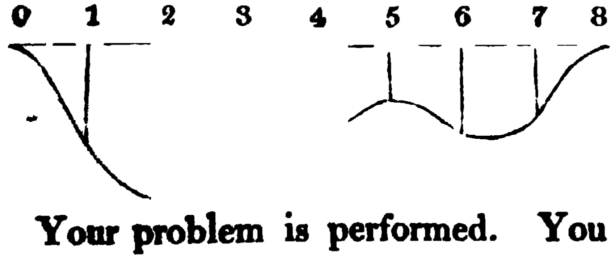
Let the perpendicular, drawn from the point 2 on your new line, be also one half of the perpendicular, which is drawn from the point 2 in your first figure.

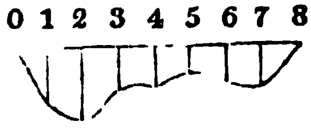
In like manner, let all the perpendiculars drawn from the various points on your second line, be exactly half of those perpendiculars, which are drawn from the corresponding points in the first figure.





Draw a curved line joining the extremities of all the perpendiculars of your new figure.





Your problem is performed. You have drawn a curve on half the scale similar to your given curve.

REMARKS ON PROBLEM XXXVIII.

This problem is very useful in Surveying, to which it may be applied in two different ways.

In the first place, it serves for laying down in a plan the figure of all irregular lines, or objects.

Supposing for instance that it were required to survey a crooked hedge; the land-surveyor would take any two convenient points in the hedge, and measure the exact distance between them in a straight line. He would also, at certain intervals, measure the perpendicular distance from this line to the opposite parts of the crooked hedge.

He would then draw a right line, on his plan, to represent the distance between the two leading points chosen in the hedge; from which, at various parts, he would also draw perpendiculars to represent the above-mentioned perpendicular distances measured upon the ground.

The right line and perpendiculars, thus laid down upon paper, being regularly drawn, on a scale of so many yards or feet to an inch as the surveyor thinks proper, and being made to correspond exactly with his previous measurements; he has only to join the extremities of all these perpendiculars, marked in his plan, by a curved line; and he will have an exact representation of the hedge, which he was required to survey; because the small curve, drawn upon paper, will be similar to the large curve formed upon the ground by the crooked hedge.

Secondly: upon this problem, that very useful and necessary art, the art of Levelling, is also principally founded.

Let us suppose, for instance, that it were necessary to level some hollow irregular ground between two hills, such as might be represented by the original curve in this problem.

The land-surveyor, who was required to perform this operation, would first, by means of his instruments, find out two convenient points on these hills, that were exactly on the same level, which may be represented by the points marked 0 and 8, in your figure. The right line in the figure, which connects these points, will therefore represent the true level or horizontal line in nature, the same as would be formed, if a string could be stretched quite tight, and on an exact level, between the points marked on the two hills. The surveyor would also measure the exact distance between these points in a right line.

He would next ascertain, by means of his levelling instruments, how many feet any particular part of the valley lay lower than the two points chosen for his first level. As the difference of height between the hills and the valley would of course vary at different parts of the low ground; he would continue taking levels, at certain intervals, between the two above-mentioned points, until he had got as many as he judged necessary.

He would then draw a right line on paper to represent the distance between his original points chosen on the two hills, from various parts of which line, he would drop perpendiculars to represent the length of his several levels.

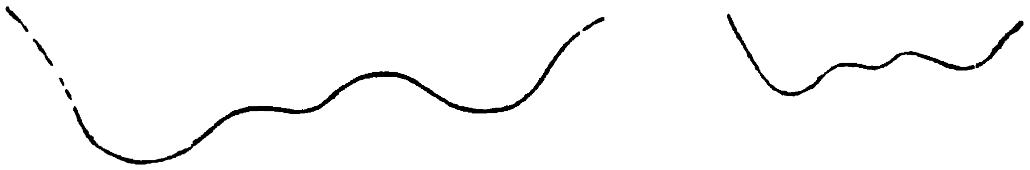
The above right line and perpendiculars being laid down upon paper, according to a regular scale of yards or feet, so as to correspond exactly with the surveyor's previous measurements and observations; he has only to join the bottom of all his perpendiculars by a curved line. This will give him a correct representation of the ground which he was required to level; because the small curve, thus delineated upon paper, will be similar to the large curve, formed in nature, by the outline of the two hills, and of the valley or low ground between them.

It is by no means necessary that a foreman of civil carpenters, masons, &c., or a non-commissioned officer in the Royal Engineer department should be able to take levels on a great scale, such as would be required in forming the plan of a canal, that was to extend a considerable number of miles. But it will be useful to you to know a little of the art of levelling, in order that you may be able to level any spot of ground on which a work or building is proposed to be erected, or to lay out the same according to such slope as your officers may direct.

The simple parts of levelling, to which I now allude, may be learned by an intelligent man, in a few days' practice.

When a finished plan is made, in which any crooked hedge or other irregular object is represented; the right line and perpendiculars, by means of which it was originally drawn, must be rubbed out as soon as the curve is marked with ink.

Rub out the right lines and perpendiculars, by means of which you drew your new curve in the present figure.

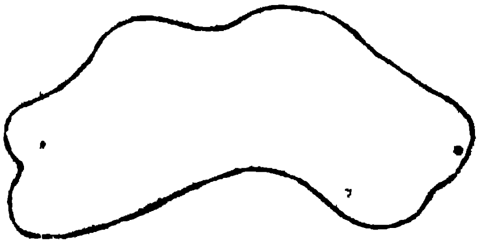


Two curved lines now only remain, which are similar to each other; the smaller curve being on half the scale of the other.

PROBLEM XXXIX.

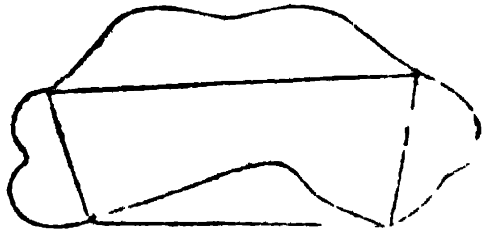
TO DRAW A FIGURE SIMILAR TO A GIVEN CURVE-LINED FIGURE.

Draw a curve-lined figure to. represent the given figure.



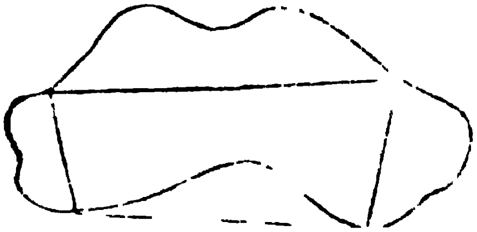
Then a certain number of points must be marked in the curve, as many as you judge convenient; and these must be joined by right lines, which will form an irregular right-lined figure, corresponding in some points with your given figure.

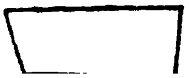
In our present figure four points may be sufficient. Mark the curve with four points, and connect these points by as many right lines, which will form a trapezium.



Let us suppose that the required figure is to be drawn on half the scale of our given figure.

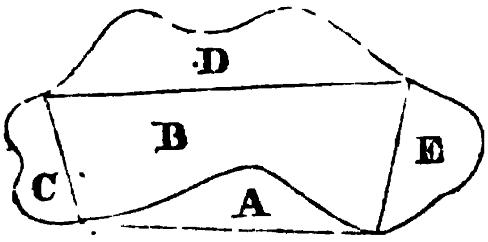
You will first make a trapezium similar to that which you have just drawn, on half the scale.

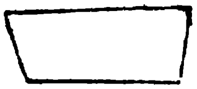




In your given figure a part of the curve divides your first trapezium into two parts. Mark one of these parts (that which is next the base) with a capital A, and the other with a capital B.

Mark also the remaining parts of your given figure by the capital letters C, D, and E.

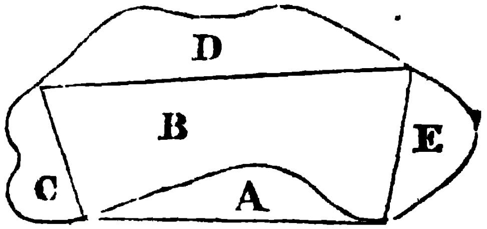




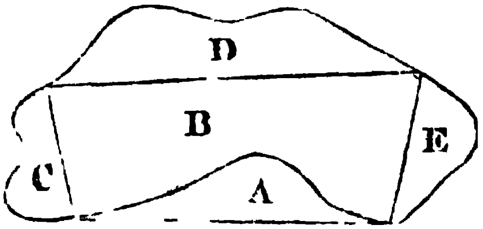
Upon the base of your small trapezium, draw a curved line similar to the curved line which bounds the figure A in the large trapezium. This must be done according to the method explained in the last problem.

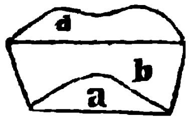
This new curved line will divide your small trapezium into two parts, which will be similar to the divisions of your large trapezium.

Mark these parts with the small letters a, b, so as to correspond with the former.



On the top of your small trapezium, draw a curved line, similar to that which bounds the figure D, in the large trapezium. This new curve will form a figure above your small trapezium, similar to the figure D. Let this be marked with a small d,



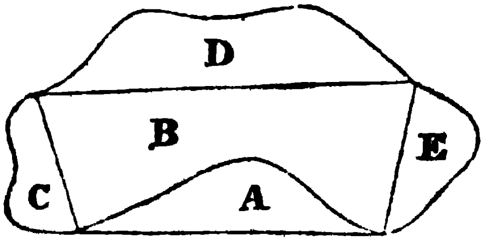


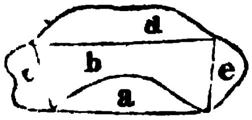
On the left side of your small trapezium, draw a curve on half the scale, similar to that which bounds the figure C.

This will form a new figure to the left of your small trapezium, similar to C. Let it be marked with a small c.

On the right side of your small trapezium, draw a curve on half the scale, similar to that which bounds the figure E.

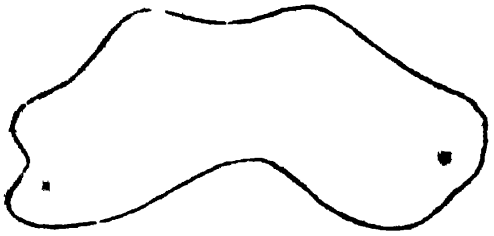
This will form a new figure similar to the figure E. Let this be marked with a small e.

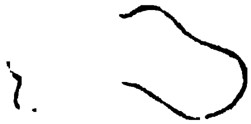




All the parts of your new figure are similar to the corresponding parts of your given figure, on half the scale.

Rub out right lines and letters in both figures.





Your problem is performed. The small curve-lined figure which remains is similar to your given figure.

REMARKS UPON THE LAST FIVE PROBLEMS.

The plan of any thing is nothing more than a figure drawn on paper, on a small scale, exactly similar to the figure of the real object, which it is intended to represent.

In like manner the model of any thing is merely a solid body on a small scale, similar to the real object or solid body, which it is intended to represent.

Hence if you understand thoroughly the foregoing problems, which teach the mode of drawing figures similar to any kind of given figures; you will find no difficulty in learning every thing else, that may be required in plan-drawing or modelling.

It is not necessary for foremen of civil artificers, or for non-commissioned officers in the Royal Engineer department, to know any of the higher and more difficult branches of drawing: the only thing required is, that they should be able to understand a drawing or model of such buildings, works, or machines, as they are generally employed in constructing.

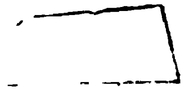
Every one knows, that almost all works of this description are laid out at right angles; and even in earth-works, where there are oblique angles in consequence of the slopes, every thing is marked by right lines: consequently the kind of plan-drawing, which you will be required to understand, depends upon the simplest of all the foregoing problems; namely, upon the being able to draw parallels and perpendiculars, and to make a triangle similar to a given triangle, or a rectangle similar to a given rectangle.

Although the knowledge of land-surveying is not by any means necessary; it will be useful to you to know how to take the plan of a single field, or of a small spot of ground, on which it may be proposed to erect a work or building. This may be done by simple measurement, without any of the mathematical instruments used by land-surveyors.

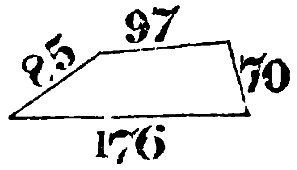
If the field is bounded by right lines, draw a rough sketch to represent the form of it as near as you can guess. This will be an irregular right-lined figure. You will next draw as many diagonals as will be necessary to divide the whole of it into triangles, according to Problem 37. Afterwards measure all the sides of your field and all the diagonals, and mark down every measurement thus found opposite to its corresponding line in the rough sketch.

From this rough sketch it will then be easy to make a regular and correct plan, by laying down the length of every line accurately upon another piece of paper, according to a scale of yards or feet.

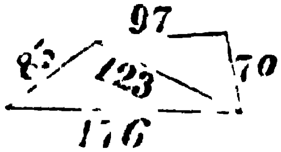
Here the Teacher will draw a trapezium upon the board.



Let us suppose for instance that I were required to take the plan of a field, shaped like the trapezium, which I have just drawn on the board. This trapezium would then represent the rough sketch of the field.



I would first measure all the four sides of the field. Supposing that the base proved to be 176 yards long: I would put down the number 176 opposite to it. Supposing that the left side of the field prove to be 85 yards long: the upper side of it to be 97 yards long; and the right side of it to be 70 yards long: I would in like manner put down the numbers 85, 97, and 70, opposite to these several sides of the sketch.



I would then measure a diagonal between two opposite angles of the field. Supposing that this diagonal proved to be 123 yards long, I would draw a line in my rough sketch to represent it; and I would mark the number 123 opposite to this line.

This divides the field, of which I am to draw the plan, into two triangles, whose sides are all known, being marked by certain numbers.

In drawing the finished plan, I should therefore only have to draw first one triangle, and then another, whose sides shall be equal to given right lines. The length of these right lines would be found by measuring 176 yards, 85 yards, and 123 yards, from a scale of yards for the first triangle. The space of 123 yards also forms one side of the second triangle. For the two remaining sides of the second triangle, I would take 97 yards, and 70 yards, from the same scale. Having constructed the two triangles by means of the above distances, I would then rub out my diagonal; and a correct plan or representation of the field would remain.

If the field, of which it might be required to take a plan, should be bounded by curved lines, it would be necessary, after drawing a rough sketch of it, to follow the rule laid down in Problem 39: dividing the whole field into portions, which would consist partly of right-lined and partly of curved-lined figures. The plan of the right-lined portions would be taken in the manner just described, by measuring all the sides and diagonals necessary: the plan of the curved portions of the field would be taken, by measuring right lines and perpendiculars, in the manner explained in Problem 38.

All the right lines and perpendiculars being marked upon the rough sketch, with the length of each opposite to it; it will then be very easy to make a correct plan, by first laying down the above dimensions accurately, according to a scale of yards or feet; after which the curve may be drawn, by joining the extremities of all the perpendiculars.

DEF. 84. When there are four lines, and the first is to the second in the same proportion, as the third is to the fourth; then the last line is called a fourth proportional to the three former.

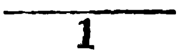
The same holds good in numbers. For instance 1 is to 2 in the same proportion as 3 to 6: therefore the number 6 is called a fourth proportional to the numbers 1, 2, and 3.

Write the words PROPORTIONAL LINES, and FOURTH PROPORTIONAL.

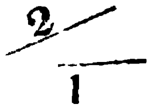
PROBLEM XL.

TO FIND A FOURTH PROPORTIONAL TO THREE GIVEN LINES.

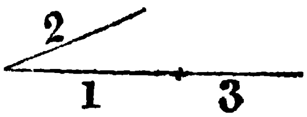
Draw a right line to represent the first given line, and mark it with the number 1.



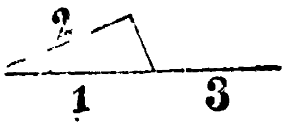
From either extremity of this line, draw another right line forming any angle with it (but an acute angle is best), to represent the second given line; and mark this new line with the number 2.



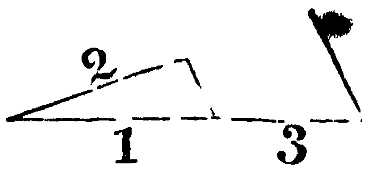
Produce the other extremity of your first given line, to any distance, and let this produced part represent the third given line. Mark it with the number 3.



Join the extremities of your first and second given lines by a right line.



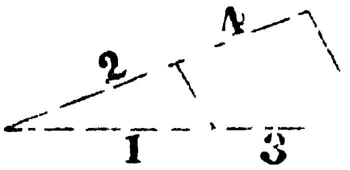
From the extremity of your third given line, draw a parallel to the last drawn line.



Produce your second given line until it meets the above parallel.

The produced part of your second given line is the fourth proportional required.

Mark it with the number 4; and your problem is performed.



REMARK I.

You have now found a fourth proportional to the three given lines, as was required.

That is to say; if the line marked 1, in your figure, is greater than the line marked 2; the line marked 3 will also be greater than the line marked 4, in the same proportion:

If the line marked 1 is equal to the line marked 2, the line marked 3 will also be equal to the line marked 4:

But if the line marked 1 is less than the line marked 2, the line marked 2 will also be less than the line marked 4, in the same proportion.

The Teacher will make the learners repeat this problem, causing them to draw their second given line in a certain proportion to the first, such as one half of it, equal to it, or double of it; which being done, he will afterwards direct them to measure the proportion that the third and fourth lines bear to each other, when the problem is performed; which will serve to illustrate the concluding remark, and give them a clear notion of the nature of proportionals.

REMARK II.

In the definition of similar figures, you will recollect, that it was observed, that all the corresponding sides of similar figures are porportional to each other.

In your present figure there are two triangles, namely a large triangle, which is equal to the whole figure, and a small triangle, which has the first and second given lines for two of its sides.

The small triangle, which forms a part of the large triangle, is exactly similar to it; and their corresponding sides are consequently proportional.

It is for this reason that the first and second lines, which form two sides of the small triangle, are proportional to the third and fourth lines, which form part of the corresponding sides of the large triangle.

DEF. 85. When there are three lines, the first of which is to the second, in the same proportion, as the second to the third; then the last line is said to be a third proportional to the two former.

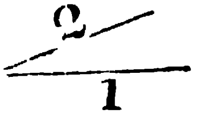
The same holds good in numbers. For instance 1 is to 3, as 3 is to 9: therefore the number 9 is called a third proportional to the numbers 1 and 3.

Write the words THIRD PROPORTIONAL.

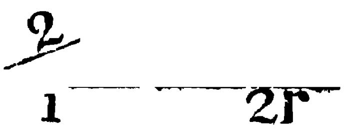
PROBLEM XLI.

TO FIND A THIRD PROPORTIONAL TO TWO GIVEN LINES.

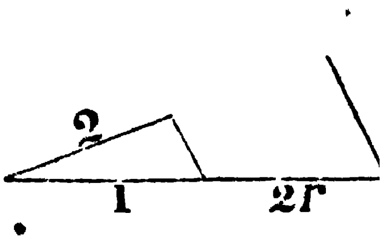
Draw two right lines forming an angle, to represent the first and second given lines, to which a third proportional is required; and mark these two lines with the numbers 1 and 2.



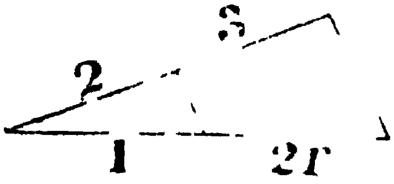
Produce the first given line, on that side which is farthest from the angular point; make the produced part equal to the second given line; and mark it with 2 r, to show that it is the second line repeated.



Join the extremities of your first and second given lines by a right line; to which you will draw a parallel, from the extremity of your second line repeated.



Produce your second given line until it meets the above parallel, and this produced part will be the third proportional required. Mark it with the number 3.



If the line marked 1 in your figure is greater than the line marked 2; the line 2 (or 2 repeated, which is the same thing) will also be greater than the line marked 3 in the same proportion:

If the line 1 is equal to the line 2; the line 2 will also be equal to the line 3:

But if the line 1 is greater than the line 2; the line 2 will also be greater than the line 3, in the same proportion.

Your problem is therefore performed; you have found a third proportional to the two given lines.

DEF. 86. When three lines are proportional to each other, like the lines in this problem; the second line is said to be a mean or middle proportional to the two others.

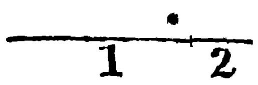
Write the words MEAN PROPORTIONAL.

PROBLEM XLII.

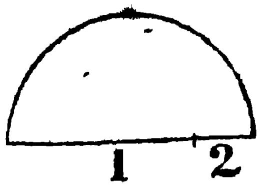
TO FIND A MEAN PROPORTIONAL TO TWO GIVEN LINES.

Draw a right line to represent the first given line; and mark it with the number 1.

Produce this line to any length, and let this produced part represent the second given line: and mark it with the number 2.

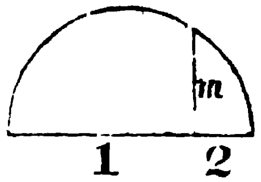


Upon the whole line thus formed describe a semicircle; the center of which will previously be found by bisecting the given line.



From the common extremity of the two given lines, raise a perpendicular meeting the circumference.

This perpendicular will be a mean proportional to the two given lines. Mark it with the letter m.



Your problem is now performed: For if the line marked 1 is greater than the line marked m; the line marked m will be greater than the line marked 2, in the same proportion:

If the line 1 is equal to the line m, the line m will be equal to the line 2:

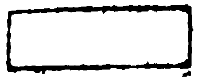
And if the line 1 is less than the line m, the line m will be less than the line 2, in the same proportion.

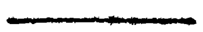
The line m is consequently a mean proportional between your two given lines, as was required.

PROBLEM XLIII.

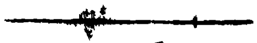
TO MAKE A SQUARE EQUAL TO A GIVEN RECTANGLE.

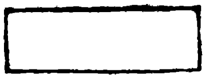
Draw a rectangle to represent the given rectangle.



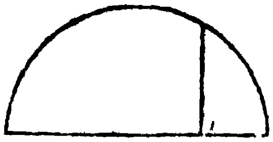
In another part of your slate, draw a right line  equal to the base of your rectangle:

Produce this line, and make the produced part equal to the height of your rectangle.

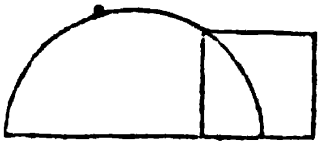




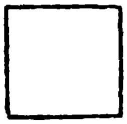
Find a mean proportional between these two lines, by describing a semicircle, &c. in the manner explained in last problem.

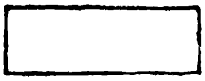


Upon this mean proportional make a square.



Rub out every thing excepting this square and the given rectangle.



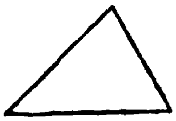


The above square will be equal to the given rectangle: your problem is therefore performed.

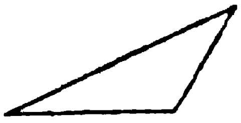
PROBLEM XLIV.

TO MAKE A RECTANGLE EQUAL TO A GIVEN TRIANGLE.

Draw a triangle to represent the given triangle.



or



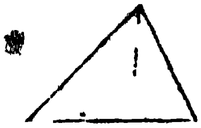
The Teacher will only use one of the above figures at a time.

From the vertex of your given triangle drop a perpendicular to the base.

If the vertex of the triangle should overhang the base, which sometimes happens, you must produce the base, and drop the perpendicular to the base produced.

Dot this perpendicular.

If the base should be produced, dot also the produced part of your base.



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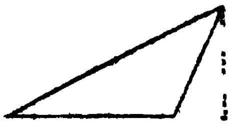
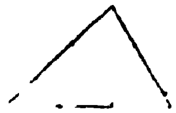


Unless the vertex of the triangle, which the Teacher gives as an example, should actually overhang; he will not read those parts of the above instructions, which relate to producing the base.

Draw a right line equal to half the base of your given triangle, to represent the base of the proposed rectangle.



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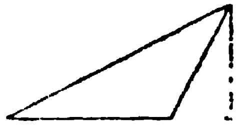
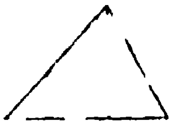


From the two extremities of this line raise perpendiculars, each equal to the altitude of your given triangle; and join the top of these perpendiculars by a right line, which will complete a rectangle.



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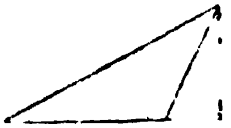
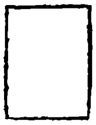




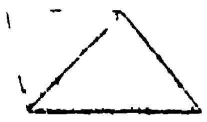
Your problem is performed: the rectangle, which you have just drawn, is equal to the given triangle.

PROBLEM XLV.

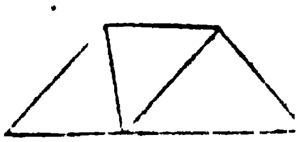
TO MAKE A TRIANGLE EQUAL TO A GIVEN QUADRILATERAL.

Draw a quadrilateral to represent the given quadrilateral.

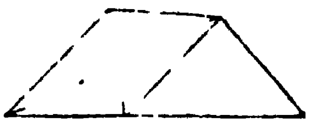
From either extremity of the base draw a diagonal across your figure, meeting one of the angles at the top of the given figure.



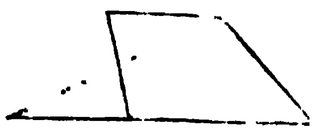
From the other angle at the top of your given figure, draw a right line parallel to this diagonal; and produce the base of your given figure till it meets the parallel.



From the extremity of the produced part of the base, draw a dotted line to the top of the diagonal.



Rub out the diagonal, and the line which was drawn parallel to it to meet the base produced.

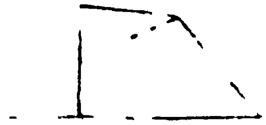


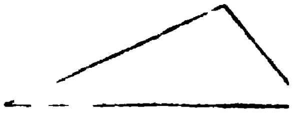
There now remains your original quadrilateral figure, with its base produced, and a dotted line.

You may observe, that the base produced, the above dotted line, and one of the sides of the given figure, form a triangle.

This triangle will be equal to the given quadrilateral figure: but, for the sake of clearness, the two figures ought to be divided.

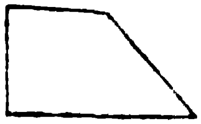
On another part of your slate you will therefore make a new triangle equal to the above triangle.

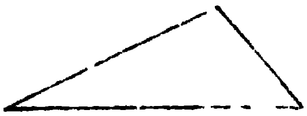




Rub out the superfluous lines attached to your given figure.

Your problem is now performed: you have drawn a triangle equal to a given quadrilateral figure.



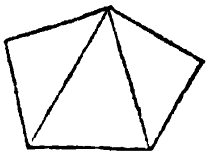


PROBLEM XLVI.

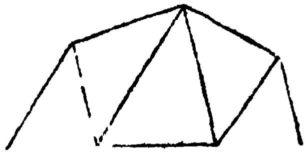
TO MAKE A TRIANGLE EQUAL TO A GIVEN PENTAGON.

Draw a pentagon to represent the given pentagon.

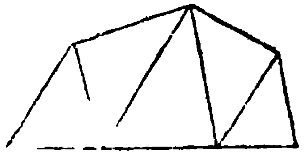
From the extremities of the base, draw two diagonals meeting in an angle at or near the top of your figure.



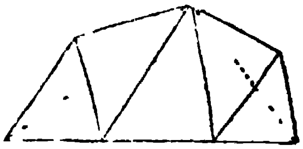
From those two angles of your given pentagon, which are immediately to the right and left of the above angle, draw lines parallel to the two nearest diagonals.



Produce the base of your given pentagon, till it meets the above parallels.

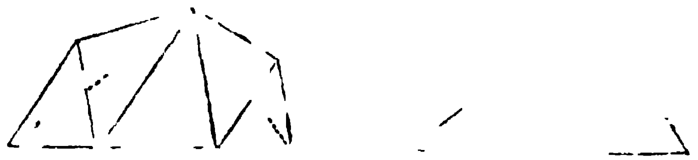


From that angle of your given figure where the diagonals meet, draw two dotted lines to the extremities of the base produced.

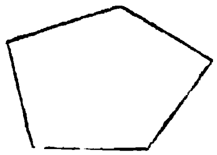


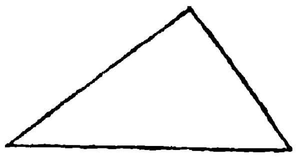
These two dotted lines and the base produced form a triangle, which will be equal to your given quadrilateral; but it is proper that the two figures should be separated.

On a different part of your slate, you will therefore make a new triangle equal to the above triangle.



Rub out the diagonals and other superfluous lines, which were added to your original figure.





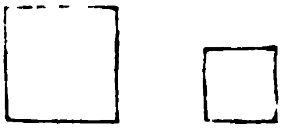
Your problem is now performed: You have drawn a triangle equal to a given pentagon.

By following the same method, used in this and in the foregoing problem, a triangle may be made, equal to any other kind of given polygon.

PROBLEM XLVII.

TO MAKE A SQUARE EQUAL TO THE SUM OF TWO GIVEN SQUARES.

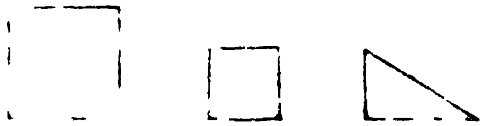
Draw two squares to represent your two given squares.



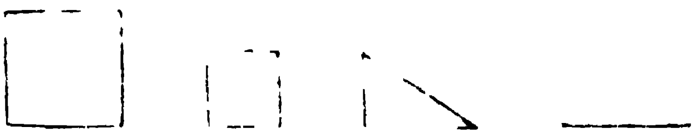
Draw a right line equal to the side of your first given square.

From one extremity of this right line raise a perpendicular equal to the side of your second given square.

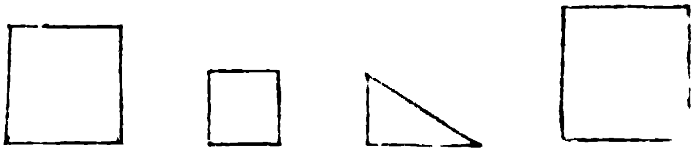
Join the extremities of these two right lines by a third right line, which will complete a right-angled triangle.



On another part of your slate draw a right line equal in length to the hypothenuse of the above triangle.

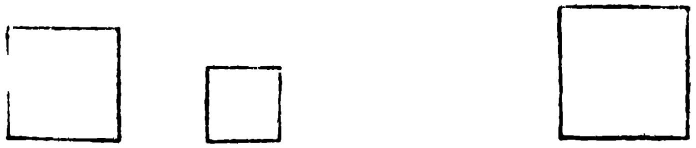


Upon the last drawn line, as a base, make a square.



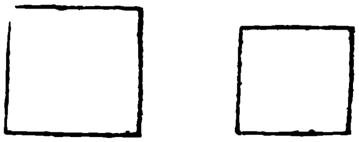
This last square will be equal to the sum of the two former squares.

Rub out your right-angled triangle, and your problem is performed: you have made a square equal to the sum of two given squares.

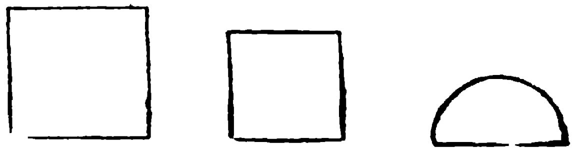


PROBLEM XLVIII.

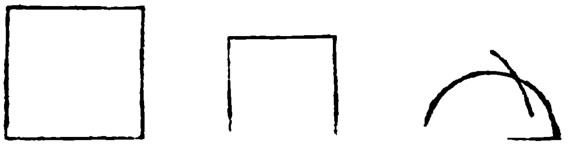
TO MAKE A SQUARE EQUAL TO THE DIFFERENCE OF TWO GIVEN SQUARES.



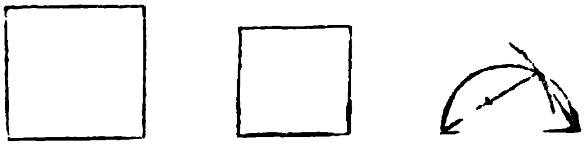
Draw two squares to represent your two given squares; observing to make one of them larger than the other; for if they were both equal, there could be no difference.



On another part of your slate draw a right line equal to the side of your largest given square; and on this right line, as a diameter, describe a semicircle.

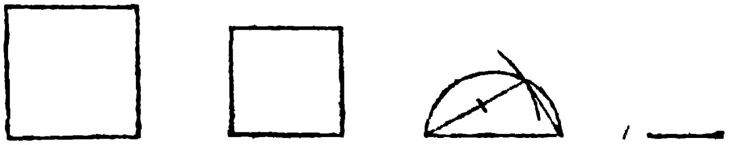


From one extremity of the diameter, as a center, with a radius equal to the side of your smallest given square, make an intersection upon the semicircumference.



From the above point of intersection, draw two right lines to the extremities of the diameter. These will form two chords, one of which will be equal to the side of your smallest given square.

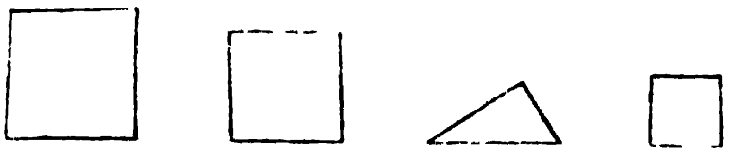
Mark this with a point.



On another part of your slate, draw a right line equal to the second chord, which is not marked.

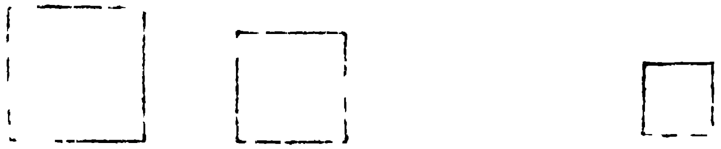
Upon this new line, as a base, make a square.

Rub out the point, which was made to distinguish one of the chords; and rub out also the circumference of your semicircle.



There now remain three squares, and a right-angled triangle, which has its three sides equal to the sides of the above squares.

Rub out the triangle.



The last drawn square will be equal to the difference of your two given squares: that is to say the largest of your two given squares is equal to the smallest of the two given squares, and to the third square put together.

Consequently your problem is performed.

DEF. 87. When any solid body is cut right in two, the figure of that part, which is cut, is called a section.

Write the word SECTION.

For instance, if any beam or spar of wood is sawed right across, so as to cut it into two pieces; observe either of these pieces, and the new end of it, that is to say, the end which has been formed by the saw, will be a section of the original timber.

If the original piece of timber was a squared beam, the section of it when cut across will therefore be a square or a rectangle; but if it was a mast or spar, the section of it will either be a circle, or some curved figure resembling a circle.

In order to illustrate the above definition, the Teacher will have two small pieces of wood, one shaped like a beam, the other like a spar, which must be made so as to separate into smaller pieces, forming various sections.

The section of a solid body may be taken in any direction, not only across, but lengthwise; not only perpendicular or parallel to the base, or to any of the sides or ends of the body; but oblique to them: so that these various sections of the same body may have very different figures.

All the sections of any body are, however, supposed to be plane superficies; that is to say, the body is supposed to be cut straight through, not in an irregular manner.

In speaking of the sections of any solid body, it is therefore usually said that the body has been cut by a plane in a certain direction.

The sections of a cone are more useful in geometry, than those of any other solid.

Write the words SECTIONS OF A CONE OR CONIC SECTIONS.

The Teacher will provide himself with wooden cones cut in various directions, in order to illustrate the following definitions.

When a cone is cut by a plane passing through the vertex, and any part of the base, the section formed will be a triangle.

When a cone is cut by a plane parallel to the base; the section will be a circle.

The triangle and circle, although they may be formed by cutting the cone, are not however usually understood when the term conic section is mentioned.

The term conic section is therefore, strictly speaking, only applied to the following figures.

DEF. 88. When a cone is cut obliquely through both sides of it, the section is an ellipse.

This figure is also sometimes, but improperly, called an oval.*

---

* Some writers use the word ellipsis in preference to ellipse.

Write the words ELLIPSE.

DEF. 89. When a cone is cut by a plane parallel to one of its sides, the section is called a parabola.

Write the word PARABOLA.

The vertex of a conic section is the point, where the cutting plane meets the side of the cone.

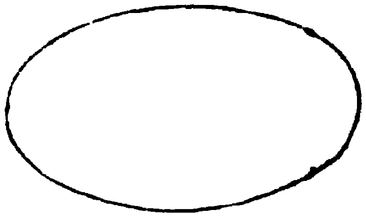
As the ellipse cuts both sides of the cone, it has two vertices.

Write the VERTICES OF AN ELLIPSE.

As the parabola cuts only one side of the cone, it has but one vertex,

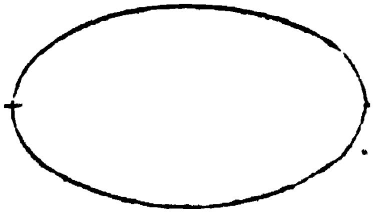
Write THE VERTEX OF A PARABOLA.

The Teacher will draw a figure resembling an ellipse on the board.



The figure which I have just drawn represents an ellipse. I will afterwards show you the method of making a regular one. In the mean time this rough sketch will serve to explain the definitions.

I will now mark the vertices of my ellipse by two points.

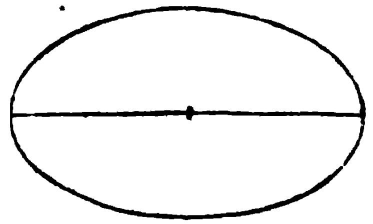


Draw each of you a figure to represent an ellipse; and mark points for the vertices of it, in the same manner as I have done.

DEF. 90. A right line drawn between the two vertices of an ellipse is called the transverse axis.

Write the words TRANSVERSE AXIS.

Draw the transverse axis of your ellipse; and mark the middle of it by a point.



The middle point of the transverse axis is the center of the ellipse.

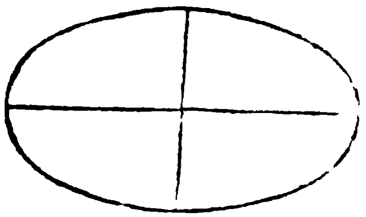
Any line drawn through the center of an ellipse and meeting the curve on both sides is called a diameter.

The diameters of an ellipse are not all equal like those of the circle, but are of different lengths.

The transverse axis of an ellipse is the longest diameter.

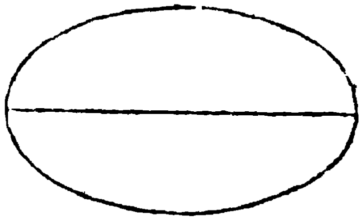
DEF. 91. The shortest diameter of an ellipse is called the conjugate axis.

Draw a diameter, or right line passing through the center of your ellipse, perpendicular to the transverse axis. This will be the shortest diameter, or conjugate axis.



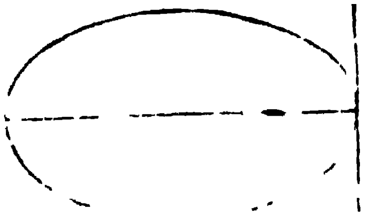
Write the words CONJUGATE AXIS.

Rub out your conjugate axis.

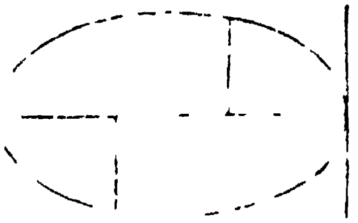


DEF. 92. An ordinate to any diameter is a line drawn parallel to a tangent, which passes through the extremity of that diameter.

Draw a tangent passing through either vertex of your ellipse, which consequently will also pass through one extremity of the transverse axis.



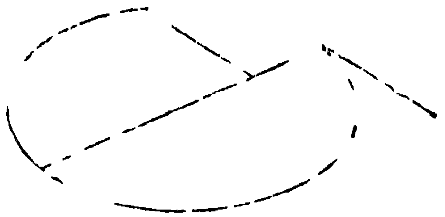
·From any points in the transverse axis, draw lines parallel to this tangent, meeting the curve on either side.



These last drawn lines are called ordinates to the transverse axis.

All the ordinates of the transverse axis are perpendicular to it, because the tangents at its extremity are perpendiculars; but in any of the oblique diameters the ordinates would not be perpendiculars; because the tangents at their extremities would not be at right angles to them.

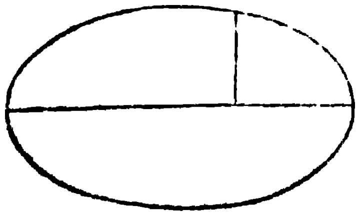
Here the Teacher will draw another ellipse, with an oblique diameter, tangent and ordinate, in order to illustrate the above observation.



Observe this new ellipse which I have just drawn. It has an oblique diameter the tangent to which is not at right angles to it. Consequently its ordinates, which by the definition must always be parallel to the tangent, cannot be perpendicular to the diameter.

The Teacher will rub out this figure, leaving the former one.

Rub out the tangents of your ellipses, and also one of your ordinates.



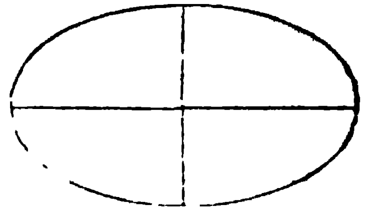
DEF. 93. Any part of the diameter of a conic section terminated by the curve is called an absciss.

Write the word ABSCISS.

For instance your present figure represents an ellipse, with its transverse axis or longest diameter, and one ordinate.

This ordinate divides the transverse axis into two parts, which are called abscisses.

Rub out your ordinate and draw the conjugate axis once more.



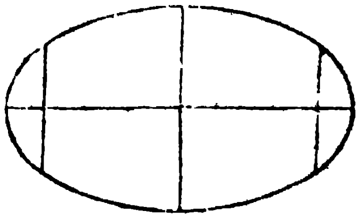
If a number of double ordinates are drawn any where between the conjugate axis and the vertex; the largest of them will be a little less than the conjugate axis, and they will diminish gradually from that size till they come to nothing at the vertex.

The Teacher will draw some double ordinates to explain this, which he will afterwards rub out.

The transverse axis being greater than the conjugate axis; as the transverse axis is to the conjugate, so will the conjugate axis be to some other line which must be smaller than the conjugate. This line will of course be a third proportional to the transverse and conjugate axes.

As the double ordinates of the ellipse are all of different lengths diminishing gradually from the length of the conjugate axis to nothing; two of them must necessarily be third proportionals to the transverse and conjugate axes; namely, one on one side, and another on the other side of the center of the ellipse.

Draw two double ordinates to represent the above third proportionals.



DEF. 94. A third proportional to the transverse and conjugate axis, is called a parameter.

Consequently the two double ordinates which you have just drawn are the two parameters of your ellipse.

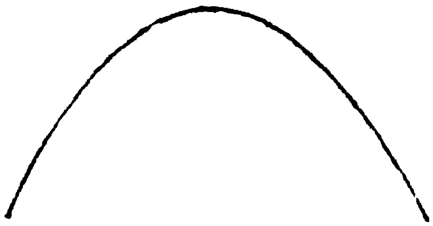
Write the words PARAMETERS OF AN ELLIPSE.

DEF. 95. The point, where either of the two parameters intersects the transverse axis, is called a focus; and in speaking of both these points together, they are called the two foci.

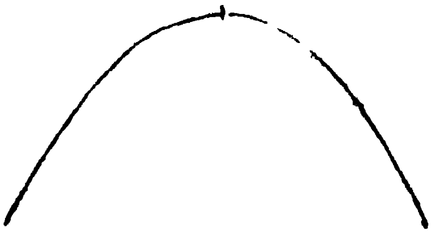
Write the words FOCUS OF AN ELLIPSE.
Write also THE TWO FOCI OF AN ELLIPSE.

Rub out your figures.

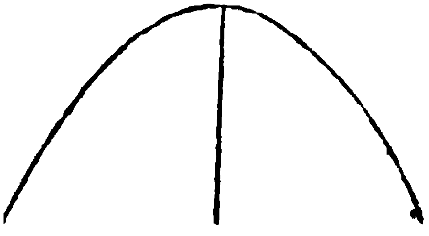
I will now draw you a sketch of a parabola; which you will copy as accurately as you can.



Mark the vertex of your parabola by a point.

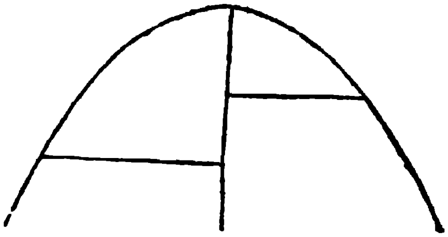


Draw the transverse axis of your parabola, that is to say from the above point or vertex draw a right line dividing the figure into two equal parts, which will consequently be equally distant from the curve on each side.



The ordinates of the parabola, like those of the ellipse, are perpendiculars drawn from any point in a diameter meeting the curve on either side.

Draw a couple of ordinates to the transverse axis of your parabola.



That part of the transverse axis, which is comprehended between the vertex and any of the ordinates, is called an absciss to that ordinate.

In short the same terms are applied to the parabola as to the ellipse: only that the parabola not being a complete figure like the ellipse, it has but one vertex, as was before observed; and, in a parabola, there is no fixed length to the transverse axis or to the two sides of the curve.

There is also this further difference, that every ordinate in an ellipse has two abscisses, whereas the ordinates in the parabola have only one absciss.

DEF. 96. When a cone is cut by a plane passing through the base and one side of it, in a direction not parallel to the opposite side, but oblique to it, and in such a manner, that the cutting plane and the opposite side, if produced, would meet some where above the vertex of the cone; the section is called a hyperbola.

Write the word HYPERBOLA.

The Teacher will produce a cone cut in this manner, and by means of a couple of rulers, one laid to the side of the cone, the other to the section, so as to meet in some point above the vertex, he will be able to illustrate the above definition.

The two rulers must be applied to that piece of the cone only which has the vertex complete, the other piece being previously removed.

REMARKS UPON CONIC SECTIONS.

The hyperbola is a figure which has never been applied to any mechanical purpose, we shall therefore take no further notice of it; but the ellipse and parabola have been found very useful in the arts.

The ellipse being a very elegant figure is often used by architects and mechanics for ornaments in buildings or furniture.

The arches of bridges have also frequently been made in the form of a half ellipse. The arches of Blackfriars-bridge for instance are elliptical.

The parabola has also been considered a very good figure for arches. All the arches of the new casemates at Dover are parabolical.

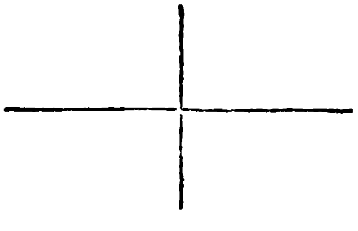
I will therefore teach you the method of drawing the ellipse, and the parabola, as you probably may, hereafter, have occasion to see these figures used.

PROBLEM XLIX.

TO DESCRIBE AN ELLIPSE WHOSE TRANSVERSE AND CONJUGATE AXES ARE OF A GIVEN LENGTH.

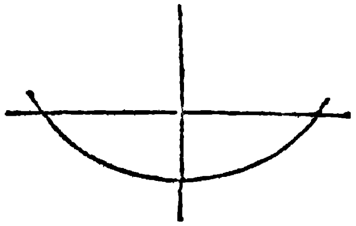
METHOD 1. By means of a string or thread.

Draw two right lines of unequal lengths, perpendicular to each other, and bisecting each other.



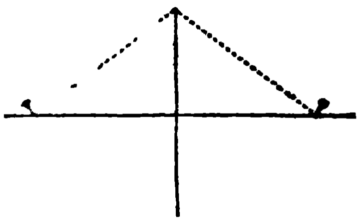
Let the longest of the above right lines represent the given transverse axis, and let the shortest of them represent the given conjugate axis, of your proposed ellipse.

From the top of the conjugate axis, with a radius equal to one half of your transverse axis, describe an arc, intersecting the transverse axis in two points.



Rub out the whole of the arc, excepting the above two points of intersection; these will be the two foci of your proposed ellipse.

From the top of the conjugate axis, draw two right lines to the foci of your ellipse; and let these two lines be dotted.



The remainder of this problem cannot be performed upon slates; you will therefore pay attention to the manner in which I shall proceed upon the board.

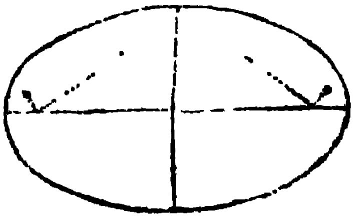
Into the two foci of my proposed ellipse I shall first drive a couple of pins or nails.

Here the Teacher will drive two pins or nails into the foci of his figure.

To these two nails I shall next fasten the two ends of a string or thread, which must be exactly equal in length to the two dotted lines; that is to say, when the thread is stretched as much as possible from the two foci or fixed points, the middle of it shall just reach the top of the conjugate axis.

The Teacher will fasten the thread accordingly.

I shall next apply a piece of chalk (or pencil) to the inside of the thread; and keeping the thread always stretched as much as possible, I shall move the chalk (or pencil) quite round.



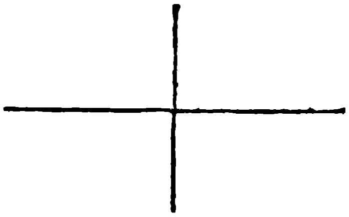
The two dotted lines in this figure, as was before explained, represent the original position of the thread: the two foci, or lower extremities of them, represent the points where the two pins or nails are driven, to which the ends of the thread are afterwards fixed. The angular point at top, where the dotted lines meet, represents the original spot where the piece of chalk (or pencil) was first placed; and the curve, which is drawn in this figure, but does not appear in the former ones, represents the curve, which would be formed by the Teacher in moving his piece of chalk (or pencil) all round, in the manner above directed.

PROBLEM XLIX.

TO DESCRIBE AN ELLIPSE, WHOSE TRANSVERSE AND CONJUGATE AXES ARE OF A GIVEN LENGTH.

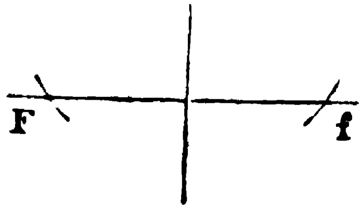
METHOD 2. By means of intersecting arcs.

Draw two right lines of unequal lengths perpendicular to each other, and bisecting each other, to represent the given transverse and conjugate axes.

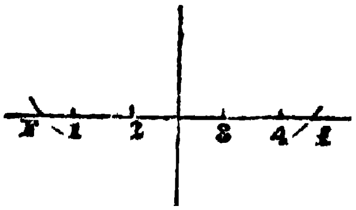


With half the transverse axis as a radius, from th conjugate axis as a center, make two intersecti transverse axis.

These two points will be the foci of your required ellipse: mark them with the letters F, f, making one a capital letter, and the other a small letter.

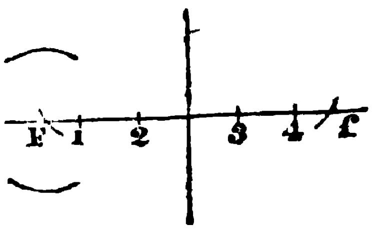


In addition to the two foci, mark any number of other points, four for instance, on the transverse axis; and number them by the figures 1, 2, 3, 4.

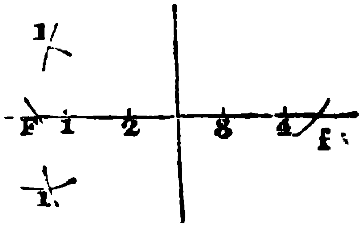


Each of these points will divide the transverse axis into two unequal parts.

With that part of the transverse axis, which is to the left of the point 1, as a radius, from the left focus, as a center, describe small arcs both above and below the transverse.

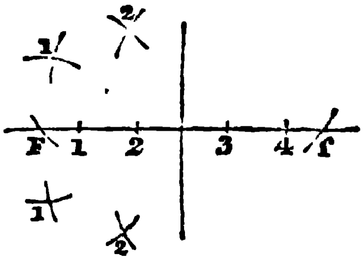


With that part of the transverse axis which is to the right of the point 1, as a radius, from the right focus, as a center, describe two small arcs, also above and below the transverse, intersecting the two former arcs; and mark the points of intersection thus found, by the figures 1 and 1, to show that they are connected with the point 1 in the transverse axis.

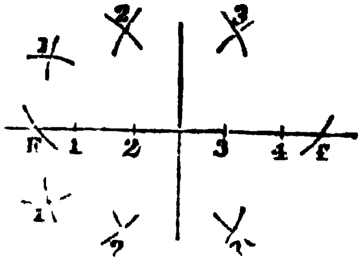


With that part of the transverse axis, which is to the left of the point 2, as a radius, from the left focus, as a center, describe two small arcs both above and below the transverse.

With that part of the transverse axis which is to the right of the point 2, as a radius, from the right focus, as a center, describe two small arcs intersecting the two last drawn arcs: and mark the two last intersections thus found, by the numbers 2 and 2, to show that they are connected with the point 2 in the transverse axis.

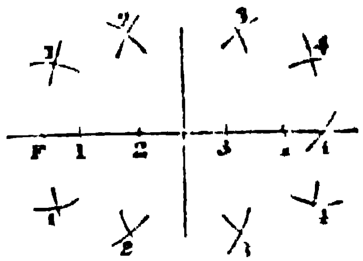


With that part of the transverse axis, which is to the left of the point 3, as a radius, from the left focus, as a center, describe two arcs both above and below the transverse axis.



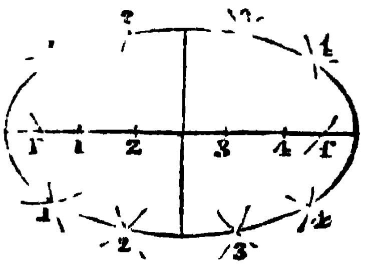
With that part of the transverse axis, which is to the right of the point 3, as a radius, from the right focus, as a center, describe two arcs intersecting the two last drawn arcs: and mark the points of intersection thus found by the figures 3 and 3, to show that they are connected with the point 3 in the transverse axis.

With that part of the transverse axis which is to the left of the point 4, as a radius, from the left focus, as a center, describe two arcs both above and below the transverse axis.



With that part of the transverse axis which is to the right of the point 4, as a radius, from the right focus, as a center, describe two arcs intersecting the two last drawn arcs: and mark the points of intersection thus found by the figures 4 and 4, to show that they are connected with the point 4 in the transverse axis.

The various intersections of the above pairs of small arcs, or of any other number of intersecting arcs, described in the same manner, will all coincide with the curve of your required ellipse.



You will therefore draw a curved line, to connect the extremities of the two given axes, and the various points of intersection of your present figure.

REMARK.

All the points of intersection, found in this manner, being true points in the circumference of the ellipse; and the remaining part of the curve being only sketched by the eye, it must be evident that the accuracy of this problem will depend entirely upon the number of parts, into which your transverse axis is originally divided.

In executing this problem, for the first time, I have only made you mark four points upon your transverse axis, because four points are as good as a thousand to explain the principle of this operation to persons of clear understanding: but in a large ellipse, four points would not produce intersections sufficiently near to each other, to enable you to draw the curve truly: six would be better; eight or ten would be better still; and in short the greater the number of points originally chosen, and the greater the number of intersecting arcs found, the more accurately may the curve be described.

In repeating this problem, the Teacher will cause the learners to mark more points than four on their transverse axis; which he will direct them to draw of such a length, that the ellipse, when finished, may be as large as the size of their instruments will conveniently permit. The number of points marked must be such as to produce intersections near each other.

When more points than four are used, after the learners have finished the intersecting arcs marked 4 *and* 4, *according to the directions above given; the Teacher will repeat the same orders that were applied to the first four points, applying them in like manner to the remaining points,* 5, 6, *&c. until he has gone through the whole number of points marked on the transverse axis.*

Each of these orders commences thus: "With that part of the transverse axis which is to the left," &c. &c. *and consists of two paragraphs.*

When the intersections corresponding to the whole of the points marked on the transverse axis are found, he will then read the concluding paragraph of this problem, as above, which commences with these words: "You will therefore draw a curved line to connect, &c. &c."

When this is done, the Teacher will give the following orders;

Rub out superfluous arcs, letters, and numeral figures, leaving only the curve and the two given axes.

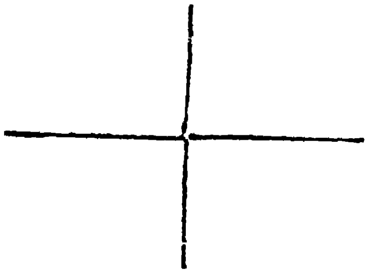
Your problem is performed. You have, by means of intersecting arcs, described an ellipse, whose transverse and conjugate axes were given.

PROBLEM XLIX.

TO DESCRIBE AN ELLIPSE, WHOSE TRANSVERSE AND CONJUGATE AXES ARE OF A GIVEN LENGTH.

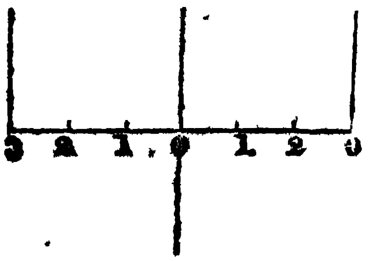
***METHOD* 3.** By means of intersecting lines.

Draw two right lines of unequal lengths, perpendicular to each other and bisecting each other, to represent the given transverse and conjugate axes of your proposed ellipse.



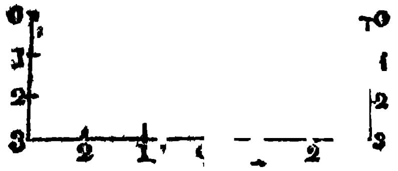
Divide the transverse axis into any even number of equal parts, six for instance, and number them regularly, both to the right and left, with the numeral figures 0, 1, 2, and 3; commencing from the center, where the figure 0 is to be marked.

From the extremities of the transverse axis, raise two perpendiculars each equal to one half of the conjugate axis in length.

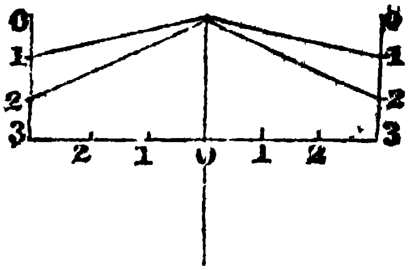


Divide each of these perpendiculars into the same number of equal parts, as there are divisions in either half of your transverse axis, which in the present instance will be three.

Number these parts regularly, on each perpendicular, with the numeral figures, 0, 1, 2, and 3, placing the figure 0 on that point which is most distant from the transverse axis.

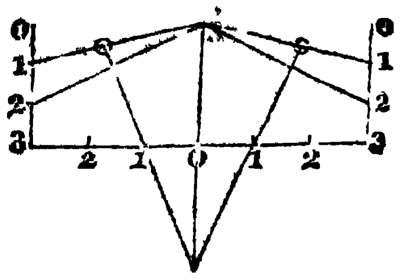


From every point on these two perpendiculars, excepting the extreme points, draw oblique lines to the nearest extremity of the conjugate axis.

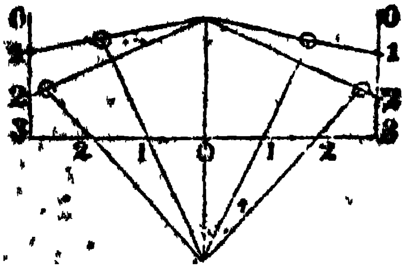


From the contrary extremity of the conjugate axis, draw oblique lines through the points marked 1 and 1 on the transverse axis; and produce them, till they meet those two of the former oblique lines, which were drawn from the corresponding points, that is to say from the points also marked 1 and 1, on the two perpendiculars.

The points of intersection, thus found, will be true points in the circumference of the required ellipse. You will therefore mark them with small circles in order to distinguish them.



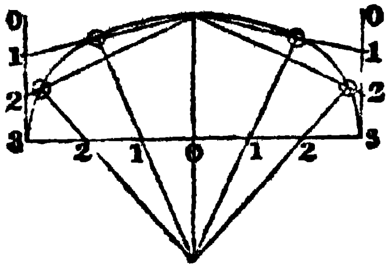
From the same extremity of the conjugate axis, last used, draw in like manner two oblique lines through the points marked 2 and 2 on the transverse axis; and produce these lines, till they meet those two of the former oblique lines, which were drawn from the corresponding points, that is to



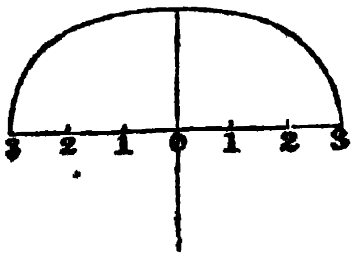
say, from the points also marked 2 and 2, on the two perpendiculars. The points of intersection, thus found, will also be true points in the circumference: mark them accordingly by two small circles.

You have now found as many points of intersection for the semicircumference of your required ellipse, as it is possible to obtain, from the number of parts into which your transverse axis is at present divided.

Connect all the points of intersection, which you have just found, with the nearest extremity of the conjugate axis, and with the two extremities of the transverse axis, by a curved line.



Rub out the two perpendiculars and all the oblique lines, leaving only the curve, the two given axes, and the points marked upon the transverse axis.



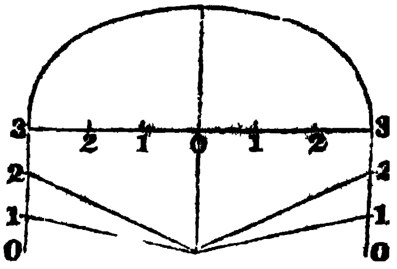
Your half ellipse is now complete. The other half of it remains to be drawn, to which exactly the same method will apply: indeed, both the half-ellipses might have been drawn at once, only that too great a number of oblique lines intersecting each other would have rendered your figure very confused.

From the extremities of your transverse axis, drop two perpendiculars, each equal to one half of the conjugate axis in length.

Divide each perpendicular into as many equal parts as there are divisions in either half of your transverse axis, which in the present instance will be three.

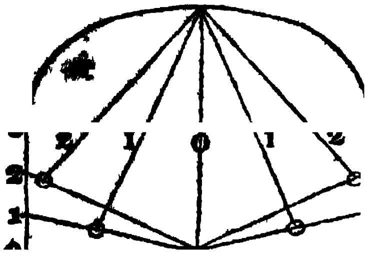
Number these parts regularly, on each perpendicular, with the numeral figures 0, 1, 2, and 3; placing the figure 0 on that point which is most distant from the transverse axis.

From every point on these two perpendiculars, excepting the extreme points, draw oblique lines to the nearest extremity of the conjugate axis.

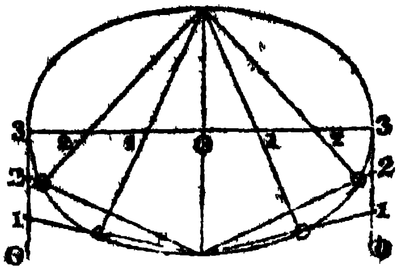


From the contrary extremity of the conjugate axis, draw also oblique lines through the points marked 1 and 1, 2 and 2, on the transverse axis; and let these be produced, until they each respectively meet one of the former oblique lines, which were drawn from the corresponding points 1 and 1, 2 and 2, on the two perpendiculars.

Mark all these points of intersection by small circles: they will be true points in the unfinished part of the circumference of your ellipse.

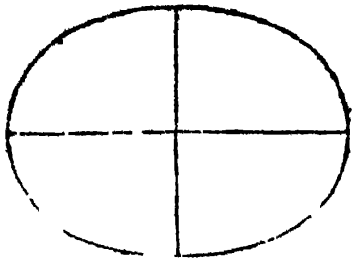


Connect all the points of intersection thus found with the nearest extremity of the conjugate, and with the two extremities of the transverse axis, by a curved line.



The curve, which you have just drawn, will be the circumference of the remaining half of your required ellipse.

Rub out superfluous lines, numeral figures, &c. leaving only this and the former curve, and the two given axes.



Your problem is now performed: you have, by means of intersecting lines, described an ellipse, whose transverse and conjugate axes were given.

REMARK.

The same remark will here apply, which was made upon the last method of describing an ellipse. The intersections found, whether by means of right lines or by means of arcs, are true points in the circumference; but all the other parts of the curve which are sketched by hand, between any two points, must be liable to error: consequently the accuracy of the operation in this method, as in the former, must entirely depend upon the number of points originally marked upon your transverse axis.

The greater the number of equal parts you pitch upon, so much the greater number of points of intersection will you be able to obtain, which will consequently become nearer to each other in the same proportion; and these intersections being all true points in the circumference, whenever the distance between them is very small, the curve may be traced with great accuracy.

Scarcely any curve except the circle can be regularly described from a center; but there are various methods of finding true points in any kind of curve; sometimes by means of perpendiculars in the manner used in problem 89, in drawing similar curves; sometimes by means of intersecting arcs or lines, according to the methods used in this problem.

In all these cases the same rule holds good: you may, by taking more pains, find as many true points in your required curve as you please; and by so doing, you may bring these points to within the distance of a hair's breadth from each other, if you think proper; so that, in afterwards drawing the outline of the curve through them, it will be almost impossible to go wrong.

Keep this remark in mind. It will not be again formally repeated, although it may hereafter apply to some of our succeeding operations.

The Teacher will cause the learners, in repeating this method of describing an ellipse, to do it on as large a scale as their slates will conveniently permit, directing them also to commence by dividing their transverse axes into more equal parts than six.

In that case, in reading the nine first paragraphs of this problem, he will make such variations in those passages, which relate to the numbers of equal parts, as may be necessary.

After he has read the ninth paragraph, and caused the learners to draw their oblique lines through the points marked 2 and 2, on the transverse axis, according to the directions therein given; he will repeat the same order contained in that paragraph as often as may be necessary, with this difference only, that instead of using the number 2, he will successively use the numbers 3, 4, 5, &c. until, by this process, he has made the learners draw oblique lines through all the points marked upon their transverse axis.

When this is done, he will go on with the next paragraph, commencing thus: "You have now found as many points of intersection," &c. &c.; *from whence he will proceed regularly, until the problem is finished.*

It was before explained to you that a semicircle means a half circle; and that the semicircumference means the half circumference.

In like manner; a half ellipse is often called a semi-ellipse.

Write the words SEMI-ELLIPSE OR HALF-ELLIPSE.

The half of the transverse axis of an ellipse is also often called the semi-transverse.

Write the words SEMI-TRANSVERSE.

And half of the conjugate axis of an ellipse is also often called the semi-conjugate.

Write the words SEMI-CONJUGATE.

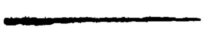
In short, whenever the term semi is put before any other term, it implies one half of the latter.

PROBLEM L.

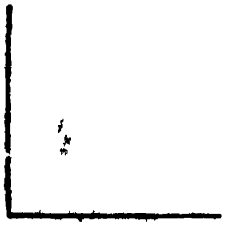
ANY ORDINATE TO THE AXIS OF A PARABOLA, AND ITS ABSCISS, BEING GIVEN, TO DRAW THE CURVE.

METHOD 1. To draw a parabola, by means of intersecting arcs.

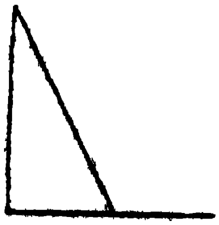
Draw a right line to represent the given ordinate.



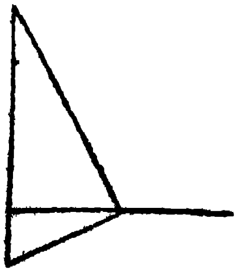
From either extremity of it, the left for instance, raise a perpendicular to represent the given abscis of your proposed parabola; that is to say, a part of the axis bounded by the vertex.



Bisect the given ordinate; and from the middle point draw an oblique line to the top of the abscis, or to the vertex of your proposed parabola.



From the same point, that is to say, from the middle of your ordinate, draw a perpendicular to the oblique line; and let this perpendicular meet the absciss produced.



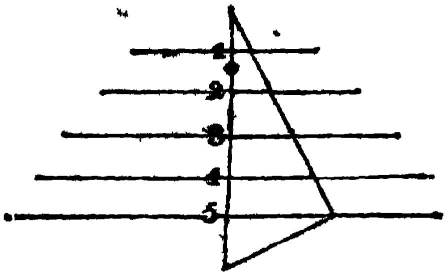
From the vertex of your proposed parabola, set off a distance upon the absciss equal to the produced part of the [illegible] and at that distance mark a point with a small circle round it [illegible] this will be the focus of your proposed parabola.

Produce your given ordinate to an equal length on the contrary side of the axis: it will then be a double ordinate.

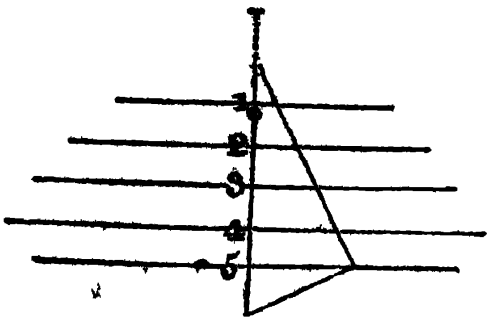
Divide the given absciss into any number of equal parts, five for instance.

Mark these parts by the numeral figures 1, 2, 3, 4, and 5, commencing from the vertex.

Draw right lines through each point of division parallel to your double ordinate.

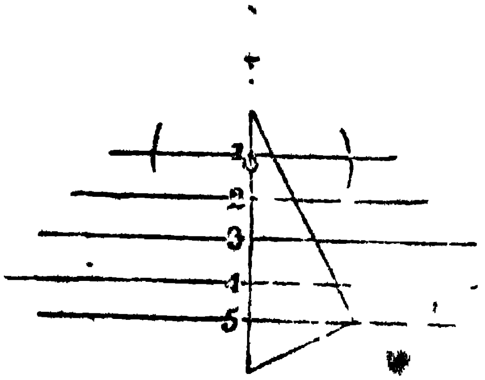


Produce the axis upwards by a dotted line, and mark a point, on the axis produced, as much above the vertex, as the focus of your proposed parabola is below it.

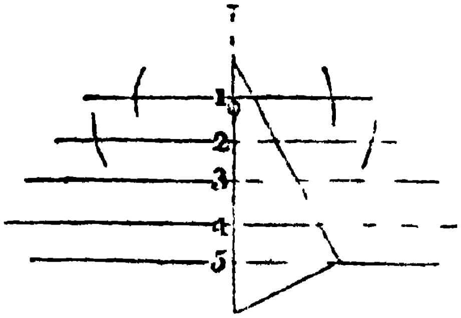


From this point the whole of the radii, for describing your small intersecting arcs, will be measured; but the arcs themselves must all be described from the focus.

Measure the distance from the point above the vertex, to the point on the absciss marked 1, and with this distance as a radius, from the focus, as a center, describe small arcs, intersecting the line which passes through the point 1, in two places.

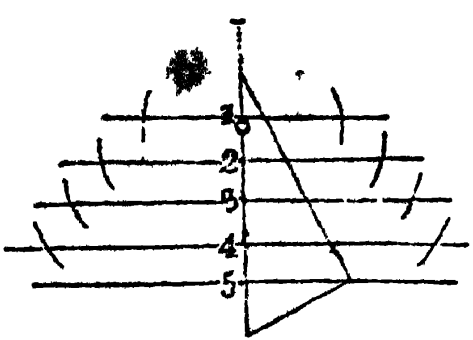


Measure the distance from the point above the vertex, to the point on the absciss marked 2, and with this distance as a radius, from the focus, as a center, describe small arcs, intersecting the line which was drawn through the point 2, in two places.



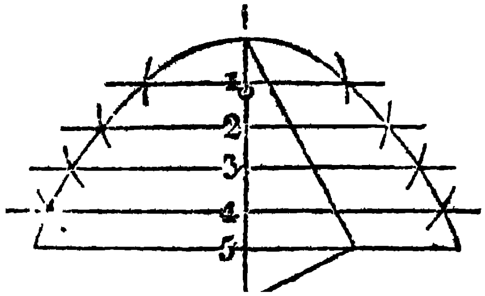
Measure the distance from the point above the vertex, to the point on the absciss marked 3, and with this distance as a radius, from the focus, as a center, describe small arcs, intersecting the line which passes through the point 3, in two places.

Measure the distance from the point above the vertex, to the point on the absciss marked 4, and with this distance as a radius, from the focus, as a center, describe small arcs, intersecting the line which passes through the point 4, in two places.

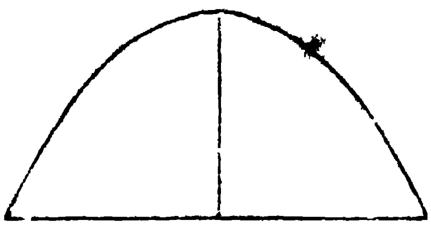


All the points of intersection, thus found, will be true points in the curve of your required parabola.

Connect all the above points of intersection, the vertex, and the extremities of the double ordinate, by a curved line.



Rub out all superfluous lines, arcs, &c. leaving only your given absciss, the double ordinate, and the curve, which you have just drawn.



Your problem is performed. The curve which you have just drawn is the parabola required.

In constructing a parabolic arch; the breadth of the arch would be represented by the double ordinate, the height of it would be represented by the absciss; and the form of the arch itself would be represented by the curve in your present figure. In cases, where parabolic arches have been used, the height of the arch has generally been made equal to half the breadth.

REMARKS ON THE PARABOLA.

In a parabola, the outline of the figure is very much curved near the vertex; whilst it is so very little curved at its lower extremities, that any small portion of that part of the curve almost approaches to a right line; as you may observe by examining your present figures.

The method, which I have just taught you, explains the general principle upon which any number of true points may be found in the parabolic curve. But it was not absolutely necessary that the points originally marked upon the given absciss, by means of which the intersections or true points were afterwards found, should have been at an equal distance from each other.

On the contrary, by reason of the variation in the form of the curve, at different parts, which was before mentioned; it is desirable that the points on the absciss should be marked closer to each other, near the vertex, than at any other part.

That you may understand this more clearly, we shall follow this new method of dividing the absciss, in performing our present problem a second time.

Rub out your figures.

In drawing your new figures let them be as large, as the size of your instruments will conveniently permit; and let the lines, which you are about to draw, for your given ordinate and absciss, be nearly equal.

Here the Teacher will go back to the commencement of the problem; and proceed as there directed, until he comes to the seventh paragraph, which relates to the divisions of the absciss: instead of reading that paragraph, he will then continue as follows.

Divide the given absciss into eight equal parts:

Subdivide the first division of it which is nearest to the vertex into four equal parts:

Subdivide the second division from the vertex into three equal parts:

And subdivide the third division from the vertex into two equal parts.

Your absciss is now divided into 14 unequal parts. Mark these parts by the numeral figures 1, 2, 3, 4, &c. &c. commencing from the vertex.

Here the Teacher will go back to the former part of the problem, and will begin at the paragraph commencing thus: " Draw right lines through each point of division," &c. *after which he will continue until he has seen the learners describe their small arcs intersecting the line which passes through the point* 4.

He will then repeat the usual order or paragraph, commencing with the words "Measure the distance from the point above the vertex," &c. *as many times as may be necessary, observing only, in place of the numbers* 1, 2, 3, *and* 4, *to substitute successively the numbers* 5, 6, 7, 8, 9, *&c. until, by this process, he has made the learners intersect all the lines which are drawn through the several points marked on the absciss.*

When this is done, he will proceed with the paragraph commencing thus: "All the points of intersection thus found will be true points," &c. *and with the three following paragraphs, which will complete the figure.*

PROBLEM L.

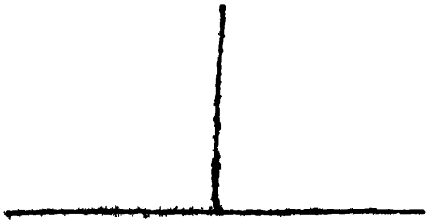
ANY ORDINATE TO THE AXIS OF A PARABOLA, AND ITS ABSCISS, BEING GIVEN, TO DRAW THE CURVE.

METHOD 2. To draw a parabola by means of intersecting lines.

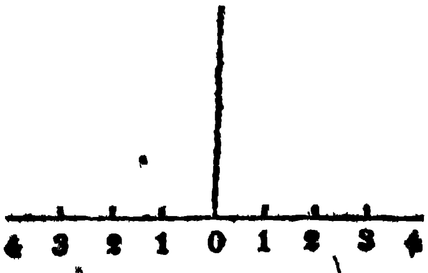
Draw a right line to represent the given ordinate.

From either extremity of it raise a perpendicular to represent the given absciss:

And convert your given ordinate into a double ordinate, by producing it to an equal length, on the other side of the absciss.

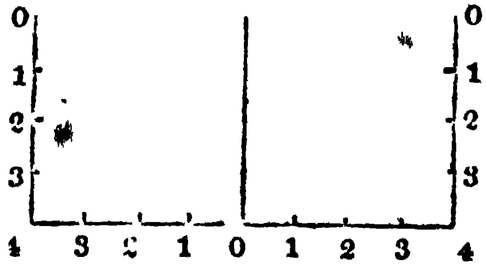


Divide each of your ordinates into any number of equal parts, four for instance; and mark them regularly, both to the right and left, by the numeral figures 1, 2, 3, and 4; commencing from the center, where you will place the figure 0.

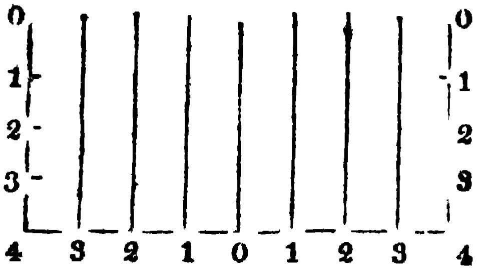


From the extremities of the double ordinate, raise perpendiculars equal to the absciss; each of which perpendicular must afterwards be divided into the same number of parts, and in the same proportion, as your ordinate; commencing from the top.

In the present case, as you divided your ordinate into four equal parts, you must divide each of your perpendiculars also into four equal parts; and number them from the top downwards with the numbers 1, 2, 3, and 4, placing the figure 0 at the top.

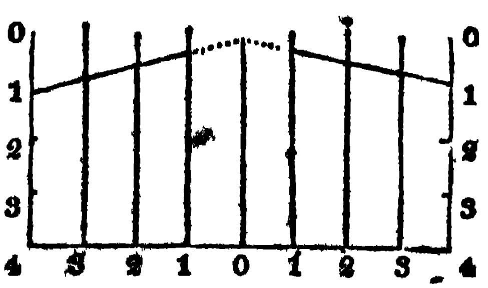


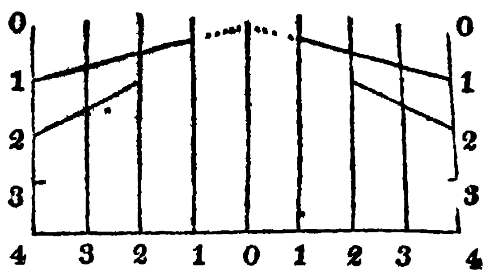
From the various points marked on the ordinates, draw right lines, parallel to the absciss or axis of your proposed parabola.



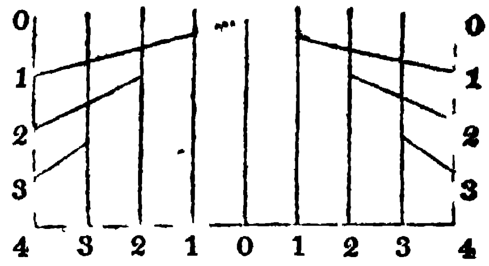
From the points marked 1 and 1, on your perpendiculars, draw oblique lines in the direction of the vertex or top of your absciss; and produce these oblique lines, until they respectively meet the corresponding lines, which were drawn from the points 1 and 1, on your double ordinate.

It is not necessary to produce these oblique lines further, after they meet the lines with which they correspond; you will, however, produce your two present oblique lines as far as the vertex, and dot them; in order that I may see, whether you have drawn them in the proper direction.





From the points marked 2 and 2, on your perpendiculars, draw oblique lines in the direction of the vertex; and produce these oblique lines until they respectively meet the corresponding lines, which were drawn from the points 2 and 2, on your double ordinate.

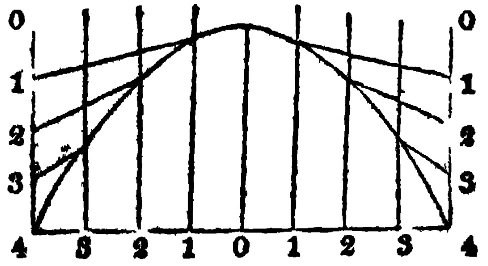


From the points marked 3 and 3, on your perpendiculars, draw oblique lines in the direction of the vertex, and produce these oblique lines, until they respectively meet the corresponding lines, which were drawn from the points 3 and 3, on your double ordinate.

If there were more divisions than four, on each perpendicular and ordinate, the Teacher would repeat the last order, or paragraph, as often as might be necessary; observing only in place of the number 3, to use successively the numbers 4, 5, 6, &c. until he had made the learners complete their oblique lines as far as the last division, but one, on their perpendiculars.

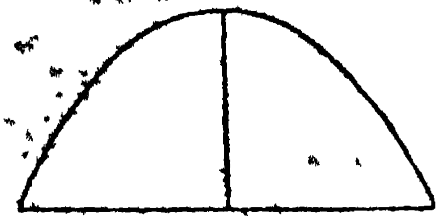
The orders, which have been already read, are sufficient for the present example; the Teacher will therefore proceed, as follows.

All the above points, where the various oblique lines respectively meet their corresponding lines, drawn from the divisions of the double ordinate, are true points in the curve of your required parabola.



You will therefore connect all the above points, the vertex or top of your given absciss, and the extremities of your double ordinate, by a curved line.

Rub out your two perpendiculars, your oblique lines, and other superfluous lines and numeral figures; leaving only the curve, which you have just drawn, the double ordinate, and its abscissa.



Your problem is performed. The curve, which you have just drawn, by means of intersecting lines, is the parabola required.

REMARK.

The same remark will here apply which I made at the conclusion of the last method of executing this problem.

It is not by any means necessary, that your ordinate and your two perpendiculars should be divided into equal parts; if they are divided in the same proportion.

On the contrary it will be an advantage to divide them unequally, provided it is done in such a manner, that the true points, which are afterwards found for describing the curve, may stand closer to each other near the vertex of the parabola, than at any other part.

I shall proceed to explain to you [illegible] manner this may be done to advantage.

Rub out your figures.

You are now to perform exactly the same operations, only that we shall commence with a greater number of divisions; you will therefore make your new figures as large as your slates can conveniently hold.

Here the Teacher will go back to the commencement of this method of drawing a parabola: and when he comes to the part respecting the divisions of the ordinates, he will order the learners to divide their ordinates into 8 equal parts, instead of four; and to place the figure 0 in the center.

He will then proceed as follows.

Subdivide the first division of each of your ordinates, commencing from the center, into four equal parts:

Subdivide the second division of each of your ordinates into three equal parts:

And subdivide the third division of each of your ordinates into two equal parts.

Number the unequal parts, into which each of your ordinates is now divided, right and left, commencing from the center, by the numeral figures 1, 2, 3, &c. as far as 14, placing the figure 0 in the center.

From the extremities of your double ordinate raise two perpendiculars, each of which must be made equal to the absciss.

Divide each of your two perpendiculars into eight equal parts.

Subdivide the upper division of each of your perpendiculars into four equal parts:

Subdivide the second division of each of your perpendiculars, reckoning from the top, into three equal parts:

And subdivide the third division of each of your perpendiculars, reckoning from the top, into two equal parts.

Number the various unequal parts into which your perpendiculars have thus been divided, by the numeral figures 1, 2, 3, &c. as far as 14, commencing from the top of each perpendicular, where you will place the figure 0.

Your ordinates and perpendiculars are now divided into the same number of unequal parts and in the same proportion, with a view to obtain more intersecting lines or true points, in that part of the curve of your proposed parabola, which will be near the vertex, than in any other part of the curve.

Having made this observation, the Teacher will go back to that paragraph of the present method of describing a parabola, which commences thus: "From the various points marked on the ordinates, draw right lines," &c. *and will go on regularly from thence, directing the learners to execute that, and the subsequent operations, until they finish the problem.*

Rub out your figures.

Having before had occasion to observe, that arches are generally built in the form of some one or other of the various curves, the nature of which has now been explained to you; I shall here introduce three problems, calculated to give you a notion of the simplest methods, usually followed in planning out arches.

These shall be called Supplementary Problems; because they do not belong to pure Geometry, but only show the manner, in which Geometry may be applied to the above branch of the art of building.

The breadth of an arch is called the span.

Write the words SPAN OF AN ARCH.

The height of an arch is called the rise.

Write the words RISE OF AN ARCH.

An arch in the form of a semicircle is called a semicircular arch.

Write the words SEMICIRCULAR ARCH.

We shall begin with this kind of arch first, it being in very common use, and one of the simplest.

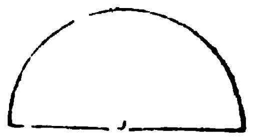
SUPPLEMENTARY PROBLEM I.

TO DRAW A SEMICIRCULAR ARCH OF A GIVEN SPAN.

Draw a right line to represent the given span of your arch.

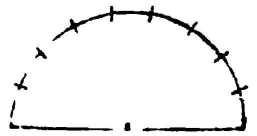
In a semicircular arch, the rise of the arch must of course be equal to half the span.

Upon this line describe a semicircle to represent the lower part of the arch: and mark the center of the semicircle.

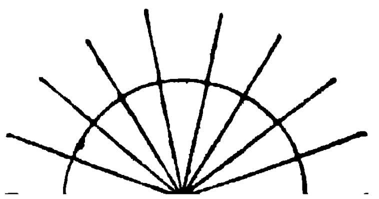


The circumference of this semicircle should next be divided into any number of odd parts, for instance into 5, 7, 9, 11, 13, &c. according to the proposed number of arch stones; which would of course depend partly upon the magnitude of your arch, and partly upon the dimensions which you might think proper to give to the stones.

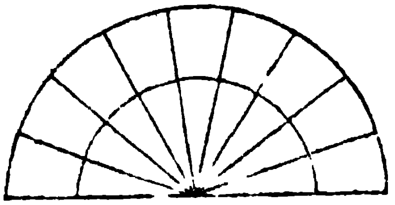
Let us suppose the present arch to be a small one, so that nine arch stones would do. Divide your semicircle therefore into nine equal parts.



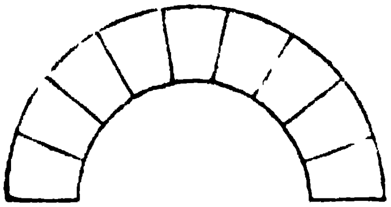
From the center of your semicircle draw radii through the above points to show the joints of the arch stones; producing the extremities of your diameter for the same reason.



From the center of your semicircle, with a longer radius, describe another semicircle, to show the top of the arch stones.



Dot the diameter of your first semicircle, and rub out all the superfluous radii.



There now remains a semicircular arch, of which the dotted line shows the span, consisting of nine arch stones, the middle and uppermost of which represents the key stone.

This rule is always used in drawing the under part of the arch, and the joints of the arch stones: but it is not necessary that all the arch stones, in any arch, should be exactly of the same length or depth, in every part, as is represented in this figure.

If the arch stones were of different lengths, it will of course be understood, that the outline of the top of them would not form a semicircle, but some other curve, or some irregular right-lined figure.

The number of arch stones is always made odd, in order that there may be a regular key-stone at the top of the arch. If the number of arch stones in this arch for instance had been eight, or any other even number, in place of nine; then, instead of having a regular key-stone at the top of the arch, there would only have been a joint between two stones, which is not approved.

Write the words KEY-STONE OF AN ARCH.

The vertex of any curve used in an arch is called the crown of the arch.

Write the words CROWN OF AN ARCH.

Sometimes an arch is formed by describing an arc of a circle less than a semicircle.

In this case the rise of the arch will necessarily be less than half the span.

An arch of this kind is usually called a segment of a circle, and it takes its name from the number of degrees, or parts of a circle, which the arc of the segment contains.

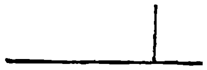
For instance, in talking of an arch of this kind, it may be said such an arch is a segment of 90 degrees; or it is a segment of 120 degrees, &c.

The manner of dividing a circle into degrees will hereafter be explained: in the mean time we shall proceed with our arches.

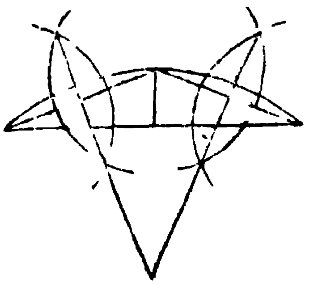
Write the word SEGMENT-ARCH.

SUPPLEMENTARY PROBLEM II.

TO DRAW A SEGMENT-ARCH OF A GIVEN SPAN AND RISE.

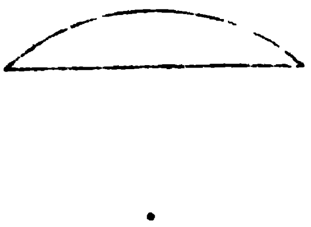


Draw a right line to represent the given span; from the middle of which you will raise a perpendicular shorter than one half of it, to represent the given rise.

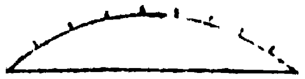


Through the three following given points, namely, the top of the perpendicular, and the two extremities of the span, describe the arc of a circle.

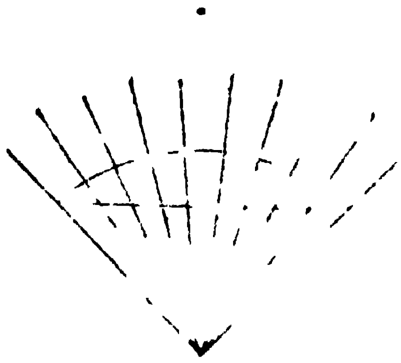
This arc will represent the lower part of your segment-arch.



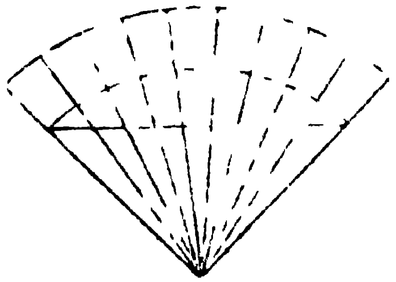
Dot the perpendicular which represents the rise of your arch; and rub out all the superfluous lines, &c. by means of which your arc was drawn, but leave the center from whence it was described.



Divide the arc into any odd number of equal parts, for instance nine, to show the bottom of the arch stones.

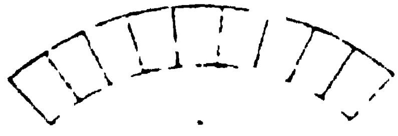


From the center of your arc, draw right lines through these points, to show the direction of the joints of the arch stones.



From the same center, with a radius longer than that of your present arc, draw a new arc meeting the two extreme or outermost right lines.

This new arc will show the form of the top of the arch stones, when they are all of equal thickness.



Rub out superfluous radii; dot your span; and the form of a complete segment-arch will remain: consequently your problem is performed.

It is to be observed, that the upper course of the two side walls or pillars which support an arch of this kind, must be cut with a bevel or skewback, to suit the form of the two extreme or outermost arch stones.

In a bridge consisting of several arches, the wall or pillar, which is between two arches, and helps to support both, is called a pier.

But the two end piers of a bridge are called abutments, to distinguish them from the others.

Write the words PIERS AND ABUTMENTS OF A BRIDGE.

The same terms also apply to the middle and end walls which support the arches of a range of casemates in a fortified place; such as are to be seen at Chatham and other garrisons.

Segment arches, such as you have just drawn, are in very common use: the whole of the arches in the new casemates at Gibraltar, for instance, have been built in this form.

They are much more suitable for a bridge than semicircular arches; because a segment-arch may be made a great deal lower than a semicircular arch of the same span, which prevents the necessity of raising the road over the bridge to an inconvenient height. Or if we suppose the segment arches and the semicircular arches to be planned exactly of the same height; then by increasing the span of your segments, you may form the bridge with much fewer arches, than if you used semicircles. Thus in a bridge composed of segment-arches you may save some masonry, and there will be less obstruction to the current of the river. Consequently a bridge of this kind will be less liable to be carried away by floods, than a bridge built with semicircular arches.

The same advantages may be obtained by using any other kind of arch whose rise is less than half the span, as for instance an elliptical arch.

Many engineers prefer an elliptical arch to a segment-arch of the same span and rise; partly because they think an elliptical arch stronger; and partly because they consider an ellipse a more beautiful figure than any arc of a circle.

Instead of using a regular ellipse, which is a very troublesome figure, it has been more common, in practice, to form arches by means of several circular segments described from various centers, and joined together in such a manner as to compose one continued curve resembling an ellipse.

A figure of this kind, which is not a regular ellipse, but very much resembles it, is called an oval.

Write the words OVAL OR FIGURE RESEMBLING AN ELLIPSE.

And the arch itself, which has just been alluded to, being composed or compounded of several segment-arches put together, may, from its nature, be called a compound arch.

Write the words COMPOUND ARCH.

Before we proceed to the method of drawing a compound arch resembling an ellipse; I must first teach you to describe the kind of curve which will be necessary.

There are various methods of drawing an oval. The more arcs that are employed in the circumference of an oval, or the greater the number of centers which are used in describing the curve, the more nearly may it be made to approach to the figure of a regular ellipse.

The method, which I shall now explain to you, is one of the simplest in common use; four centers only being required, which is the smallest number by which an oval can be described.

PROBLEM LI.

BY MEANS OF FOUR CENTERS, TO DESCRIBE AN OVAL, WHOSE TRANSVERSE AND CONJUGATE AXES ARE OF A GIVEN LENGTH.

In the succeeding problem, the very same operations here used in respect to an oval will be applied to an arch, with this difference; that wherever the word transverse axis is here used, the span of the arch will be understood in the next problem: where half the conjugate axis is here used, the rise of the arch will be understood in the next problem: and where the term oval is here used, the arch will be understood in the next problem.

The Teacher has before him, in the following paragraphs, *orders equally suited to this problem, or to the commencement of the next; he will, therefore, in reading the same, select those words only, which apply to that particular problem which he wishes to teach.*

Those words or directions, which apply to the present problem, will always be placed either above or before the corresponding words or directions, which apply to the next problem.

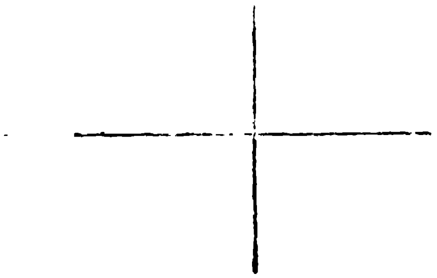
Draw a right line to represent the
given { transverse axis.
{ span of your arch.

Draw a perpendicular shorter than the former line, and let it and the former line mutually bisect each other.

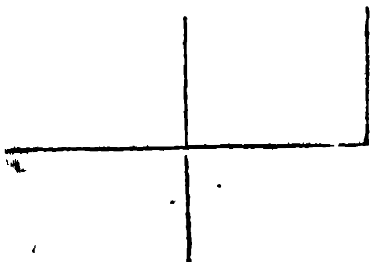
This perpendicular will represent the given conjugate axis.

Bisect it, and from the middle point raise a perpendicular shorter than half the line, to represent the given rise of your arch.

Produce the rise of your arch downwards to an equal length.

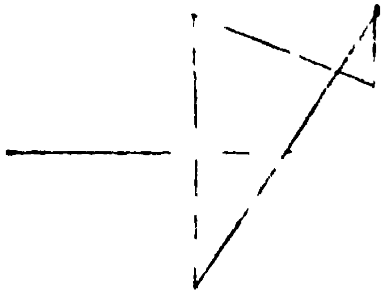


From either extremity, for instance from the right extremity,
of the { transverse axis } raise a perpendicular equal to
{ span of your arch }
the { semiconjugate.
{ rise of your arch.



Bisect this perpendicular, and from the middle of it draw an oblique line to the top of the $\left\{\begin{array}{l}\text{conjugate axis.}\\ \text{rise of your arch.}\end{array}\right.$

From the top of the perpendicular, draw a second oblique line to the bottom of the $\left\{\begin{array}{l}\text{conjugate axis.}\\ \text{rise of your arch produced.}\end{array}\right.$

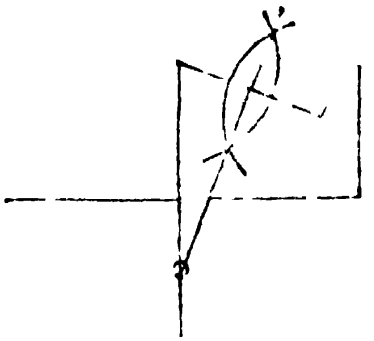


This second oblique line will of course intersect the former oblique line.

Rub out the whole of this second oblique line, excepting the above point of intersection.

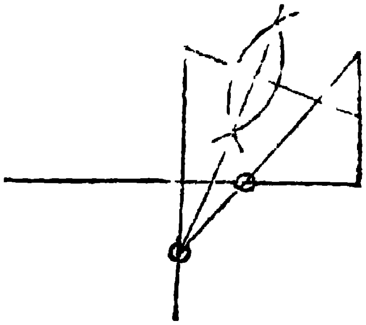
Bisect that part of the former oblique line, which is contained between this point and the top of the $\left\{\begin{array}{l}\text{conjugate axis}\\ \text{rise of your arch}\end{array}\right\}$; and from the point of bisection, draw a perpendicular downwards to intersect the $\left\{\begin{array}{l}\text{conjugate axis.}\\ \text{rise of your arch produced.}\end{array}\right.$

Sometimes it will be impossible for these lines to meet without being further produced. In that case you will produce the $\left\{\begin{array}{l}\text{conjugate axis}\\ \text{rise of your arch further}\end{array}\right\}$ downwards.



Mark the point of intersection by a small circle, to denote that it is to be one of the centers for describing a part of the curve of your proposed $\left\{\begin{array}{l}\text{oval.}\\ \text{compound arch.}\end{array}\right.$

This point shall be called the lower center.



From the lower center, draw a right line to the top of the perpendicular, and mark, in like manner, the point where this line cuts the { transverse axis } by
{ span of your arch }
a small circle.

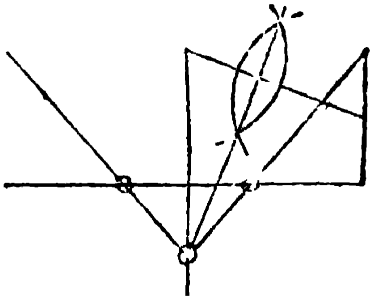
This point will be a second center for describing part of the curve of your proposed { oval.
{ compound arch.

From its position it shall be called the right center.

Take the distance between the right center, and the middle of the { transverse axis } , in your compasses; and mark a point,
{ span of your arch }
at an equal distance from the middle, on the contrary side of the { transverse axis.
{ span of your arch.

This point will be a third center for describing a part of the curve of your proposed { oval.
{ compound arch.

Mark it also by a small circle; and let it be called the left center.



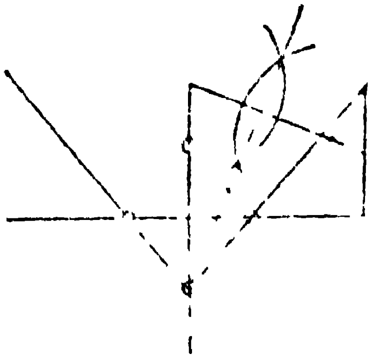
From the lower center of your figure, draw a right line through the left center, and produce it upwards about as far as the last drawn line.

The following directions apply only to the required oval, which forms the subject of the present problem.

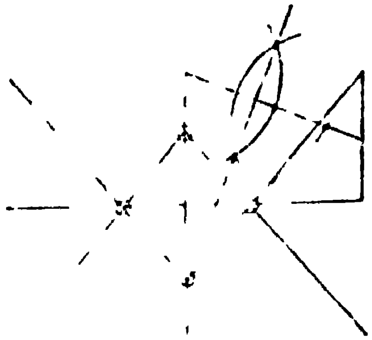
When the compound arch is to be drawn, the Teacher, after having gone thus far, will, therefore, proceed to the next problem; and continue giving the further directions therein contained.

Take the distance between the lower center, and the middle of the conjugate axis, in your compasses; and set off an equal distance from the middle, on the contrary side of your conjugate axis, or conjugate axis produced, if necessary.

Mark this new point also with a small circle: it will be the fourth or upper center of your required oval.

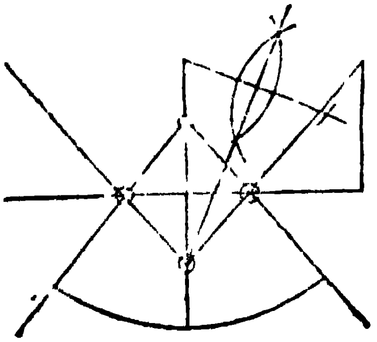


From your upper center, draw two oblique lines through the right and left centers.

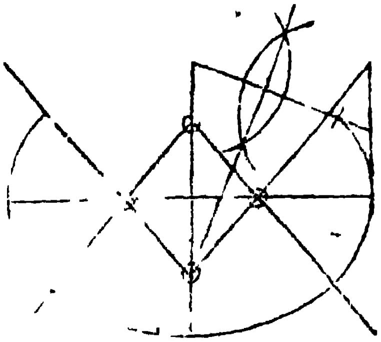


From your upper center, with a radius extending as far as the bottom of the conjugate axis, describe an arc; and let this arc be bounded by the two lines, which were drawn from the same point through your right and left centers.

This arc will be a part of the curve of your required oval.

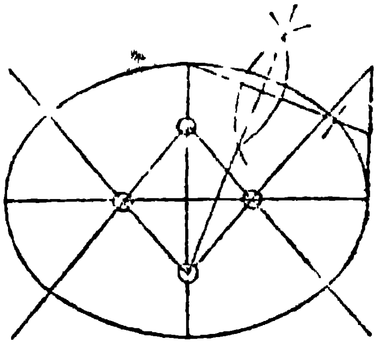


From your right and left centers, with a radius extending from each as far as the nearest extremity of your transverse axis, describe arcs outwards; and let them be bounded by the two pairs of lines, which were drawn through the same respective points, from the upper and lower centers.



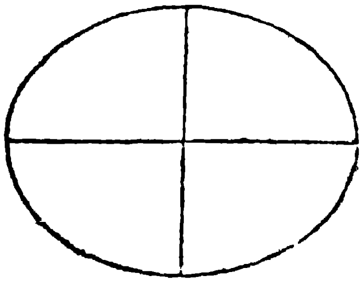
These two new arcs will also be a part of the curve of your required figure; and if you have performed the above operations accurately, they will coincide with the former arc, in such a manner, that the three together will form one continued curved line.

From your lower center, with a radius extending as far as the top of the conjugate axis, describe an arc; and let this arc be bounded by the two oblique lines, which were drawn from the same point, through the right and left centers.



This last described arc will also be a part of the curve of your required figure; and if your operations have been performed accurately, its extremities will exactly coincide with the nearest extremities of the two former arcs, which were described from the right and left centers; in such a manner that all the four arcs put together will form one continued curve-lined figure; which, as you may perceive, is exactly like an ellipse.

Rub out all superfluous lines, &c. leaving nothing except your two given axes, and the above four arcs.



Your problem is now performed. You have, by means of four centers, described an oval, or figure resembling an ellipse, whose conjugate and transverse axes were of a given length.

I shall next proceed to explain to you the method in which this problem may be applied to the art of building arches.

SUPPLEMENTARY PROBLEM III.

BY MEANS OF THREE CENTERS, TO DRAW A COMPOUND ARCH, OF A GIVEN SPAN AND RISE, RESEMBLING AN ELLIPSE.

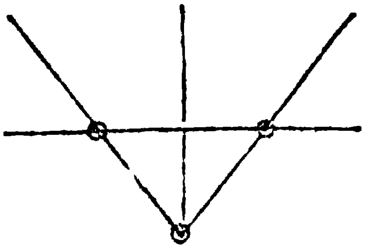
This problem is performed by exactly the same rule as the last, observing that the span of the arch, in this problem, will correspond with the given transverse axis in the former; and the rise of the arch, in this problem, will correspond with the upper semi-conjugate axis in the former problem.*

In this problem, three centers only are required, these being sufficient to describe the upper half of the oval, which of course forms a complete arch.

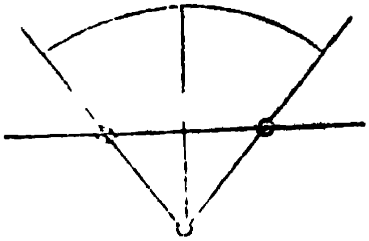
Having made this preliminary observation, the Teacher will go back to the commencement of the last problem, and give such part of the directions therein contained as apply to the object of this. He will stop at the particular place, in that problem, where he finds himself directed so to do; from whence he will immediately return to the present part of the book, and will proceed as follows.

---

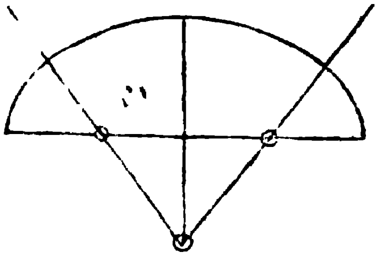
* In the figures for this problem, it has been judged convenient, to make the proportion between the span and rise of the arch, different from that which the transverse axis and the semi-conjugate bear to each other, in the last problem. Consequently, although the figures in both are drawn exactly by the same rule, the curve of the arch in this problem is not equal and similar to the half oval of the former problem; which would have been the case, if the above-mentioned proportions had been equal.



Rub out the perpendicular, the bisecting line, and that oblique line, which was drawn from the top of the rise of your arch.



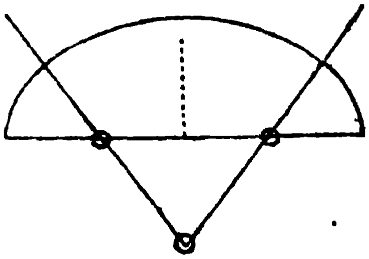
From your lower center, with a radius extending as far as the top of the rise of your arch, describe an arc; which must be bounded by the two oblique lines that were drawn from the same point.



From the right and left centers, with a radius extending from each as far as the nearest extremity of the span, describe arcs outwards, each of which must be bounded by one of the oblique lines on one side, and by the nearest extremity of the span, on the other.

If the former part of your operations has been performed accurately, the three arcs which you have just described, will coincide; in such a manner as to form one continued curve, resembling the semicircumference of an ellipse.

This curve will represent the outline of your required compound arch.



Dot the rise of your arch; and rub out the produced part of the same line.

You must next consider what will be the most convenient dimensions for your arch stones; after which you will divide your curve accordingly.

Let us suppose that our present arch is rather a small one, so that nine arch stones would be enough for the upper segment of it; you will, therefore, divide the arc of your upper segment, that is to say, the arc which was described from the lower center, into nine equal parts.

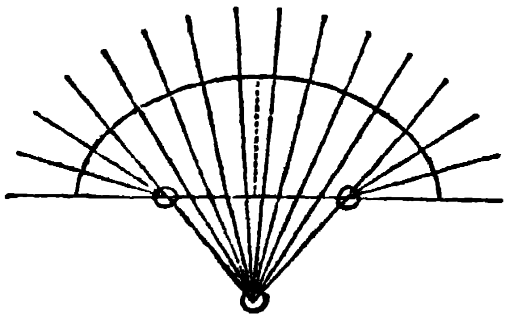
It will next be necessary to mark the arch stones of the two smaller segments. They ought to be exactly, or as nearly as possible, of the same size as those of the upper segment, in order that the whole arch may have a regular and handsome appearance; but it is of no consequence, whether the number of arch stones in each of the smaller segments is odd or even.

Divide each of the two smaller segments into any convenient number of equal parts, no matter whether odd or even; but let them be equal, or nearly equal, to the former divisions of your upper segments.

In my figure, which I have drawn upon the board, I found *(Here the Teacher will name some number, which has proved most convenient, for instance)* three to be a convenient number of equal parts for each of my small arcs; and have divided them accordingly: but, as it is not likely that your figures should have been drawn exactly similar to mine, you are not obliged to follow my example in choosing the same number of equal parts, but may divide your small arcs in any other proportion, which you think will agree better with the general rule just laid down.

Each of the three segments forming your compound arch, must next be completed individually; in the same manner, as if it were a separate segment-arch, unconnected with the others. You will therefore proceed as follows.

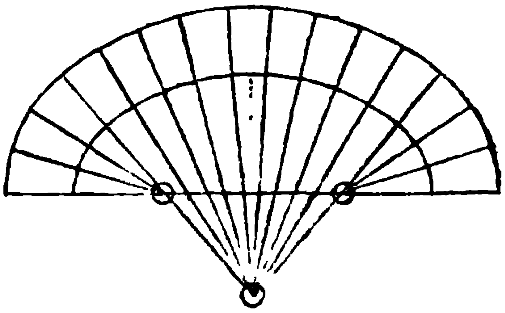
From each of your three centers, draw right lines through the points marked on their respective arcs, in order to show the joints of the arch stones; and produce the extremities of the span for the same purpose.



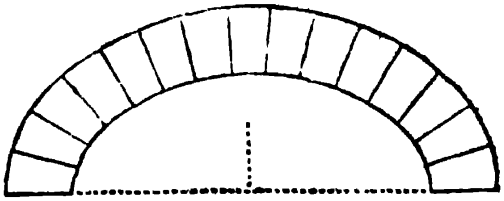
From each of the three centers of your figure, with longer radii than were used in describing the former arcs, three new arcs must next be described parallel to the former, and bounded by the same lines which bounded the former.

Let these new radii be of such a length, that the three new arcs, like the former, shall coincide so as to form one continued curve.

Choose radii, and describe arcs accordingly.



Rub out all the right lines which are drawn from your three centers to the lower curve: rub out also your three centers: and dot the span of your arch.



Your problem is now performed. You have, by means of three centers, drawn a compound arch, of a given span and rise, resembling an ellipse.

REMARKS ON THE SUPPLEMENTARY PROBLEMS.

A foolish notion generally prevails amongst young beginners in Geometry, before they are sufficiently advanced to be able to see the object of what they are learning. They are often apt to fancy, that the definitions are merely an useless string of hard words without meaning, and that even most of the problems are likely to be of little or no service to them.

It was with a view to prevent you from being misled by such erroneous notions, that the above three supplementary problems were introduced. They may serve merely as one example, out of a thousand others, which I might have selected, in order to illustrate the great and extensive utility of Practical Geometry. This example must be convincing to any person of good understanding.

Every one will allow, that there is scarcely any performance of mechanical skill, which is more useful to the public, more elegant in its appearance, or which requires greater knowledge and ingenuity to plan and execute well, than a fine bridge over some great river; such as Blackfriars and Westminster bridges, for instance, over the river Thames at London.

Yet, as far as regards the arches of such grand works, they are generally drawn according to the above simple methods, which you have just learned. And from the advantage of having gone through this course, you would find it equally easy to understand the plan of the centering or frame-work of any great arch; or the plan of the roof of a first-rate building. For Geometry, as I said before, is not merely useful in one branch of the mechanical arts; but applies equally to them all: so much so, that nothing can be done without it.

It is true that there are many very ingenious artificers and mechanics, as I before observed, who may be capable of executing difficult pieces of work; without having learned even the most common definitions in Geometry. But all the knowledge of such men being derived solely from practice; whenever their own experience stops short, they must generally come to a stand. On the contrary, a man, who has learned Geometry regularly, according to good principles, will be able to comprehend the plans and descriptions of works, which he may never have had it in his power to see; so that he can profit not only by his own practice or experience, but by that of others · and in consequence of his superior knowledge, he may even understand the nature of such works much better than ignorant persons, who may actually have assisted in the execution of them.

With respect to the various operations, in the foregoing problems, now that you have done them, you must allow that they are sufficiently simple: but perhaps you may conceive, that a man might understand and be able to perform the whole of the practical operations, which you have hitherto gone through, without having been obliged to learn so many definitions. This is perfectly true. I could have taught you the same problems with half the number of definitions; but then this would have been a very great drawback from your further improvement. Even if you knew every problem quite perfect, you could never turn your practical knowledge to proper advantage, unless you also knew the definitions, which are the regular terms of the art; because, without them, you would not be able to understand the most easy and common books on geometry and mechanics; in which these terms are constantly used.

Any person, in short, who knows the problems but not the definitions, may be compared to a man of some knowledge, turned adrift in a foreign country, without understanding the language. In such a case, it is evident that his knowledge would be of very little use to him, for he could neither explain himself to others, nor derive any benefit from their instructions. In like manner, the definitions are the language of Geometry; and must be learned.

A principal object of the system, upon which this course is conducted, is to discover such amongst you, as have shown most attention; and which can only be done by the method of taking places, and by having examinations from time to time. But let no man despair because he sees others more ready in their answers, or because he finds himself unable to get to the head of a seat.

There are various degrees of ability. Some excel in one thing, others in another; but in every branch, diligence and perseverance will lead to certain improvement. And although those who learn slowly labour under some disadvantages at first; it often happens, that they not only become as perfect in the end, as others who make greater progress at the first outset; but that they even retain the knowledge which they have acquired much longer.

The remark, which I shall now make, will apply to the whole of you. I am sensible, that it will be difficult, indeed, almost impossible, for any of you to remember off hand all the definitions and problems. You are not, however, to suffer yourselves to be discouraged on that account. You may easily conceive, that no man, however learned, or however well he may understand any subject, can carry the whole of his knowledge in his head. He must have his books and memorandums to refer to, without which he can no more give an immediate answer upon every question, that he may have studied, than a carpenter can work without his tools. For instance, the clergyman cannot always go through his sermon word for word, exactly as he wrote it, without occasionally looking at his notes. The same holds good in Geometry, as well as in every thing else.

It is therefore by no means necessary or expected of you, that you should always and constantly be able to perform the whole of the problems, or to answer to all the definitions, which you have gone through. It will be perfectly sufficient, if you can recall them to your recollection, and understand them, on looking into some book of Geometry, when you find occasion.

I shall now remind you of an observation which I before made. Although I consider it essential that you should go through the whole of this course regularly; in order that you may be able to improve yourselves by having recourse to useful books hereafter; and also that you may be equal to the execution of any of the more intricate and difficult operations, which may sometimes occur in the Royal Engineer department; yet, generally speaking, the plans of those works in which you are most likely to be employed, may be drawn according to some few of the very simplest of the problems you have learned, which no man who has gone through this course is likely ever to forget.

Having made these remarks, which I recommend to your serious attention, I shall now proceed to teach you the method of making scales of various kinds.

PROBLEM LII.

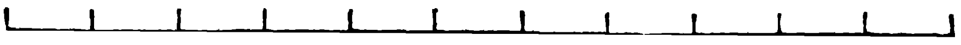
TO MAKE PLANE SCALES.

EXAMPLE 1. To make a scale of one foot to an inch.

It will be understood that the figures of scales, herein given, not being of the proper size to correspond with their respective titles, are merely intended to guide the Teacher.

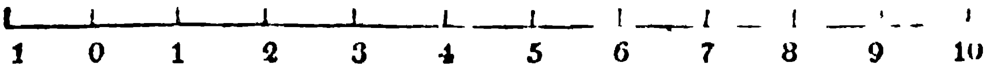
Draw a right line, and set off one inch eleven times upon it: or you may draw a right line eleven inches long, and divide it into eleven equal parts, which is the same thing.

In order to make the points of division more conspicuous, raise small perpendiculars from each of them, and also from the two extremities of the line.



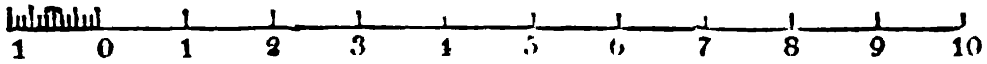
This right line is to be your scale; and according to the given proportion, each of the equal parts into which it is divided will of course represent one foot.

Under the left extremity of your line, place the numeral figure 1; and from thence, under each point of division, place successively the figures 0, 1, 2, 3, 4, 5, &c. as far as 10.



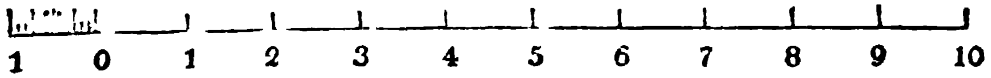
Subdivide the left division into twelve equal parts to represent inches; and let every third subdivision be marked by raising a small perpendicular, whilst the others must be marked by points only.

In addition to its small perpendicular, let the middle subdivision of the whole, also, have two dots over it, to render it more conspicuous.



Over your right line, write, in small letters, the words SCALE OF ONE FOOT TO AN INCH.

Scale of one foot to an inch.



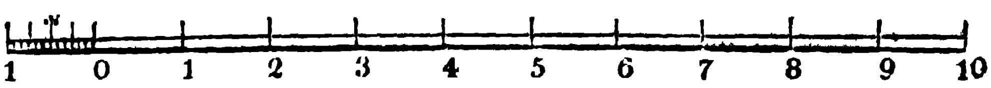
Your problem is performed. You have made a scale of one foot to an inch, as was required.

Sometimes, in a finished plan, the length of a plane scale is marked with a double line; and the lower line of the two is made thicker than the other. This makes the scale look rather handsomer; but is of little use in any other respect.

I will show you how this is done.

Here the Teacher will complete his scale, according to the method just described.

Scale of one foot to an inch.



REMARKS ON EXAMPLE I.

By the scale which you have just drawn, you may measure any number of feet and inches, not exceeding eleven feet, by one opening of your compasses.

This is done by placing the right leg of your compasses on that point of the large divisions, which corresponds with the number of feet required. If there are no inches specified, then the left leg of your compasses must be extended as far as the point marked with the figure 0.

For instance, in measuring five feet, I would place the right leg of my compasses on the point 5, and the other leg on the point 0.

But if there are any number of inches also to be measured, in addition to a specified number of feet; then, as far as regards the feet, the same rule will be followed in placing the right leg of the compasses; but the other leg must be extended beyond the point 0, until it comprehends as many of the subdivisions as there are inches to be measured.

For instance, in measuring four feet six inches, I place the right leg of my compasses on the point 4, and I extend the left leg beyond the point 0, till it covers six of the subdivisions: consequently it will stand on the middle point of subdivision which is marked by two dots.

After explaining these rules upon the board, the Teacher will *exercise the learners in setting off distances, not exceeding eleven feet, by their scales, taking care to make the whole of them set off the same distances at once, and examining the correctness of their respective performances, successively, before he allows them to attempt a new measurement. These distances must be marked on right lines, which he will order them to draw for the purpose.*

When the Teacher conceives that the learners are perfect in all measurements under eleven feet, he will proceed as follows.

By your present scales, no distances above eleven feet can be measured by one opening of your compasses. In any plan in which much longer measurements are likely to be often required, it would, therefore, be desirable to make the scale longer. This is easily done, by following exactly the same method. For instance, if I wished to make my scale capable of measuring any distance under twenty-one feet at once, I would first produce it ten inches further to the right.

The Teacher will exemplify, upon the board, this, and the following operations, necessary for completing a scale of twenty-one feet.

I would next divide this produced part into ten equal parts, of an inch each, and mark them by raising small perpendiculars.

I would lastly continue the numeration of my scale, by adding the numeral figures 11, 12, 13, 14, &c. up to 20.

Now you see, that if I wish to take any distance under 21 feet, for instance 20 feet 9 inches; I can do it by one measurement.

In the same manner, you may easily conceive, that a scale may be produced to any length: but, in general, a short or middle-sized scale is preferable; it being of no use to have a scale longer than suits the size of your instruments; because a pair of compasses, when the legs of it are too widely extended, cannot measure a distance accurately.

When your scale is shorter than the distance which you wish to measure, you will of course be obliged to set off the required distance by two or three measurements.

Here the Teacher will rub out the produced part of his scale.

I have now again reduced my scale to the same state as yours; and consequently have lost the power of measuring more than eleven feet at once.

I shall first show you how to set off fifteen feet six inches.

The Teacher will exemplify as he proceeds.

I shall draw a right line, upon which the distance is to be marked.

Upon this right line, I first set off ten feet as a part of the distance; and at the extremity of the ten feet, I set off five feet six inches.

Ten feet and five feet six inches, added together, make fifteen feet six inches: consequently the required operation is performed.

In this manner all distances under 21 feet may be measured.

Supposing that the proposed distance exceeded twenty-one feet, it would then require more than two measurements.

For instance, if I wished to measure twenty-four feet nine inches from my present scale, I would first take ten feet in my compasses and set it off twice running, which would make twenty feet; after which I would take four feet nine inches, and set it off at the end of the former measurements. This would complete my required distance, because two tens and four feet nine inches are twenty-four feet nine inches.

In like manner, [*Here the Teacher will not exemplify.*] supposing I wanted to set off a distance of ninety-eight feet upon a plan; I would take ten feet in my compasses, and set off this distance nine times; after which I would take eight feet, and set it off once. This would complete the required distance, because the nine tens are ninety, to which add eight, and the whole makes ninety-eight.

I shall now see, whether you understand the method of setting off distances from your scales, by more than one measurement.

Here the Teacher will exercise the learners in setting off distances, not exceeding twenty-one feet, by two measurements.

The Teacher will also recollect to exercise the learners in making such measurements from their scales, as he may judge necessary, after each of the succeeding examples.

PROBLEM LII.

TO MAKE PLANE SCALES.

EXAMPLE 2. To make a scale of two feet to an inch.

Allowing two feet to every inch, one foot will, of course, be represented by half an inch.

You will therefore draw a right line, and set off half an inch eleven times upon it; or you may draw a line five inches and a half long, and divide it into eleven equal parts, which is the same thing.

The figure inserted after the first paragraph of example 1, *will also serve as a guide for the figure necessary here: and the remainder of the operations in this example will be exactly the same* *as in example* 1; *to which the Teacher will refer, and give the same directions therein contained, commencing from the second paragraph, at the words,* "This right line," &c. *observing only that where the phrase* "one foot to an inch" *occurs in the former example, he must read* "two feet to an inch" *in this.*

The scale, which you have now drawn, being only half as long as the former, it will generally be convenient to extend it so far as to render it capable of taking in any distance not exceeding 21 feet, at one measurement.

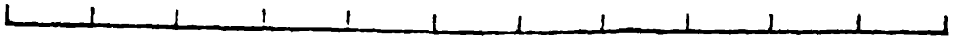
Here the Teacher will cause the learners to extend their scales accordingly, by producing them five inches more to the right; and upon this produced part setting off ten new divisions equal to their former ones, which must be numbered regularly 11, 12, 13, 14, *&c. as far as* 20.

PROBLEM LII.

TO MAKE PLANE SCALES.

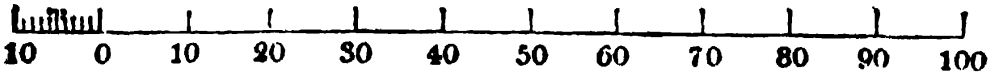
EXAMPLE 3. To make a scale of 10 feet to an inch.

Draw a right line, and set off one inch eleven times upon it; or draw a line eleven inches long, and divide it into eleven equal parts, which is the same thing: and raise small perpendiculars from each point of division, as also from the extremities of the line.



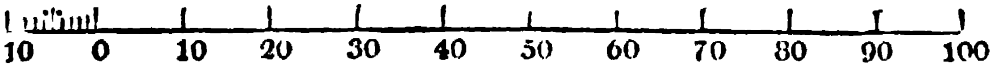
Subdivide the left-hand division into ten equal parts: and mark the middle subdivision by a small perpendicular and two dots.

Each of your large divisions will represent ten feet, whilst each of the subdivisions at the left of your line will represent one foot. You will therefore place the number 10 under the left extremity of your line; and from thence, under each large division, place successively the figure 0, and the numbers 10, 20, 30, 40, 50, &c. as far as 100.



Over your line, write the words SCALE OF 10 FEET TO AN INCH; and your problem is performed.

Scale of 10 *feet to an inch.*



PROBLEM LII.

TO MAKE PLANE SCALES.

EXAMPLE 4. To make a scale of one hundred yards or feet to an inch.

Let us suppose that a scale of yards is first to be made.

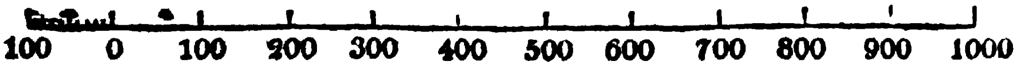
The Teacher will here read the two first paragraphs of Example 3, *which apply also to the present example. He will then proceed as follows.*

Each of your large divisions will represent 100 yards, whilst each of the subdivisions at the left of the line will represent ten yards.

You will therefore place the number 100 under the left extremity of the line, and from thence, under each large division, place successively the figures 0, and the numbers 100, 200, 300, &c. in regular order as far as 1000.

Over your line write the words SCALE OF 100 YARDS TO AN INCH; and your problem is performed.

Scale of 100 *yards to an inch.*



Rub out the word yards, and write feet in place of it in the title of your scale.

Your scale now represents a scale of feet, 100 to an inch.

GENERAL REMARKS ON PROBLEM LII.

Scales of one foot to an inch, made according to the first example, or of two feet to an inch according to the second example, or scales made in some intermediate proportion between these two, are generally used in modelling, or in plans of any nice piece of workmanship, the parts of which require to be well explained in detail.

For instance, if I were required to make the model of a pontoon, or of a traversing platform, I might perhaps choose a scale of one foot to an inch.

But if I were required to make the model of a town-gate and draw-bridge, for a fortified place, such as should show their position and proportional dimensions, in respect to the adjacent works, I should probably choose a scale of two feet to an inch, as being more convenient.

It can seldom be expedient, either in a plan or model, to use a larger scale than one foot to an inch; because the smallest dimensions that are necessary, in any piece of mechanism, may be sufficiently marked in a scale of this size.

I have, however, sometimes seen models made on a scale of half a foot to an inch. Such a scale being one sixth of the real size of the object it is intended to represent, which is a very large proportion indeed, can scarcely ever be necessary, except for the use of persons who are totally inexperienced in that particular branch of workmanship, which it is intended to explain.

On the other hand, a smaller scale than two feet to an inch woul not be sufficiently clear to explain properly the small dimensions in any nice piece of workmanship.

A scale of one foot to an inch may also be called a scale of one inch to a foot: and a scale of two feet to an inch may also be called a scale of half an inch to a foot, the proportions being the same.

It is common amongst artificers, who use these scales for models, or for working plans, to distinguish them in this manner; and for shortness' sake, they often call the former a one-inch scale; and the latter a half-inch scale.

They also frequently use an intermediate scale between these two, of three quarters of an inch to a foot, which they call a three-quarter scale.

In like manner, a scale of half a foot to an inch, may also be called a scale of two inches to a foot, and, for shortness, would be called by artificers a two-inch scale.

If you understand properly the Examples, which you have already gone through, you will find no difficulty in making a three-quarter scale or a two-inch scale for modelling.

Instead of using indiscriminately the phrase, "*Scale of so many feet to an inch,*" or, "*Scale of so many inches to a foot;*" it is much better to reject the latter expression altogether. Because if you use only the term "*Scale of so many feet to an inch,*" you may commence with a scale of one foot to an inch, and go on by regular gradations, such as five feet to an inch, ten feet to an inch, &c. By this means, after a little practice, you will always recollect what kind of scale is best adapted for the various purposes to which scales may be applied. Whereas if you confound the titles of your scales by applying different terms to the same scale, it will perplex the memory, and every time you want to use a new scale, you will be obliged to make fresh calculations and trials; so that your former experience in scale-making will be of little service to you.

I have already mentioned the scales most convenient for models, or detailed plans of nice pieces of mechanism; they vary from half a foot to an inch, to two feet to an inch.

A scale of five or six feet to an inch, is good for explaining the nature of wooden bridges, roofs, centers of arches, and other pieces of carpentry. It is also very convenient for plans, sections, and elevations or front views of small buildings, being large enough to show the various parts of columns, and other decorations or ornaments, used in architecture.

This scale is likewise well adapted for explaining the sections or profiles of field works, such as parallels, approaches, batteries, &c

A scale of eight or ten feet to an inch is convenient for drawing buildings, or pieces of carpentry, such as have before been mentioned.

These scales are also good for the plans of batteries, field powder magazines, redoubts, and other field works.

A much smaller scale would not explain clearly the various parts of a battery, &c to persons who did not previously understand the nature of such works. But for large redoubts or detached field works, in some cases, a scale of 15 feet to an inch might be preferable.

Scales of from 15 to 20 or 25 feet to an inch are useful for plans, sections, and elevations of very large pieces of architecture of which such scales will give the general effect very well, but cannot detail the various parts.

For instance, such scales would answer well for the elevation or view of a fine bridge over a large river. If the bridge had few arches, a scale of 15 feet to an inch would be preferable ; but if it was very extensive, such as Blackfriars or Westminster bridges, then a scale of 20 or 25 feet to an inch would be more convenient.

Scales of from 15 to 30 feet to an inch are useful for sections of the ramparts, ditches, and other works of a regular fortress. A much smaller scale than 30 feet to an inch would not be sufficiently clear ; and larger than 15 feet to an inch can scarcely be necessary, except only for the sections of mines and countermines. For this last purpose 10 feet to an inch may perhaps be preferable.

A scale of 100 yards to an inch is useful for the plan of a city, fortress, or estate.

For instance, if I wished to draw a plan of Chatham, or Plymouth Lines, with part of the country round them, I should choose this as a convenient scale ; it being large enough to express every thing clearly.

But if, for any particular reason, I wished to have some part of my general plan more detailed, I might then choose a scale of fifty yards to an inch. It can seldom be necessary to use a larger scale than this, except for small field works, such as have before been noticed.

If the city, fortress, &c. of which I wished to have a plan, was very extensive ; a smaller scale than 100 yards to an inch might be convenient.

For instance, if I was required to draw a plan of the fortifications of Malta, which inclose a space of several miles in extent, I would then choose a scale of 200 yards to an inch.

In like manner, if I wished to have a plan comprehending not only Chatham Lines, but the various other forts, and works connected with them, together with the cities of Rochester and Chatham; and the villages of Stroud and Upnor, on the other side of the Medway; I would then choose a scale of 200 yards to an inch in preference.

As a general rule I would prefer this scale, whenever the ground to be included in the plan exceeded three or four square miles.

The scales, therefore, which are most convenient for the plans of cities, fortresses, &c. vary from 50 to 200 yards to an inch. Any scale smaller than 200 yards to an inch would not be sufficiently clear to explain the nature of the works of a fortress. Consequently when a general plan is on a small scale, it is proper to have separate plans of particular works, on a larger scale.

Whenever it is required to use a scale smaller than 200 yards to an inch; it is then more common and more convenient, instead of a scale of yards, to make a scale of so many miles or parts of a mile to an inch.

For instance, if I wanted to use a scale of 300 yards to an inch, or thereabouts; this distance being only a few yards more than one sixth part of a mile; I should make my scale in the latter proportion in preference.

One sixth part of a mile to an inch is a very good proportion for the survey of an estate; or for the plan of the position of an army, or field of battle. It is the scale generally used by surveyors, both civil and military.

One third of a mile to an inch, or one quarter of a mile to an inch, are also sufficiently large and clear scales for similar purposes; if the ground to be surveyed is an open country.

Half a mile to an inch, or even one mile to an inch, are sufficiently large to show all the roads, bye-roads, and principal farms, and country houses.

When smaller scales than these are used, the general features of a county, province, or large tract of country, may be understood; but the particular nature of any position or small portion of it, cannot be explained, without a reference to more detailed maps or plans on some of the former scales.

It is more usual in the titles of scales for maps, to say so many inches or parts of an inch to a mile; than to say so many miles or parts of a mile to an inch.

For instance, the various scales above mentioned, may be styled scales of six inches to a mile, of three inches to a mile, of two inches to a mile, of one inch and a half to a mile, or of one inch to a mile.

This is the best method for scales of miles, and no inconvenience can arise from it, because where miles are concerned, you are not likely to be confused, although you may have been accustomed to follow a contrary arrangement, in respect to the titles of scales of yards or feet.

It is a general rule in scale-making that the numeration of the scale shall not commence from the beginning of the line, but from the right extremity of the first or left hand division, where the figure 0 is placed.

Also, that if the large divisions of the scale represent units, the subdivisions shall represent the next smaller denomination of the same measure. For instance, in your first and second examples, where each of the large divisions represented one foot only, the subdivisions were made to represent inches.

By the same rule, if you had drawn a scale of miles, of which the large divisions were numbered from 0 to 1, 2, 3, 4, 5, &c.; then the subdivisions ought to represent quarters of miles, if the scale was small; but if the scale was rather large, smaller subdivisions than these might be introduced, to represent furlongs; each of which, as you may probably know, is one eighth part, or half a quarter of a mile.

But if the large divisions of any scale represent tens, then the subdivisions must represent units of the same measure, and if the large divisions of any scale represent hundreds, then the subdivisions must represent tens of the same measure.

This is the rule which has been followed in all the above examples of scale-making. If you had divided and numbered your scales in any other manner, you would have found it impossible to measure any distance off hand without previously adding two numbers together, or subtracting one number from another. Calculations of this kind are troublesome, and may lead to error.

In plane scales, such as you have hitherto drawn, there is no method of measuring any distance which falls between two subdivisions except by guess; but, in general, the subdivisions are made so small, that plane scales are quite accurate enough for all common purposes. There is however a more accurate kind of scale called a diagonal scale, which I shall now teach you to construct.

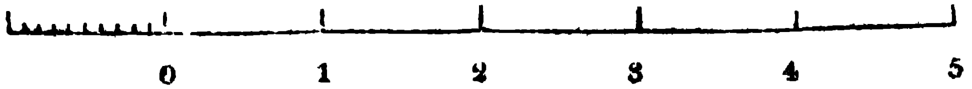
Write the words DIAGONAL SCALES.

PROBLEM LIII.

TO MAKE A DIAGONAL SCALE.

Draw a line twelve inches long: divide it into six equal parts; and subdivide the first division into ten equal parts.

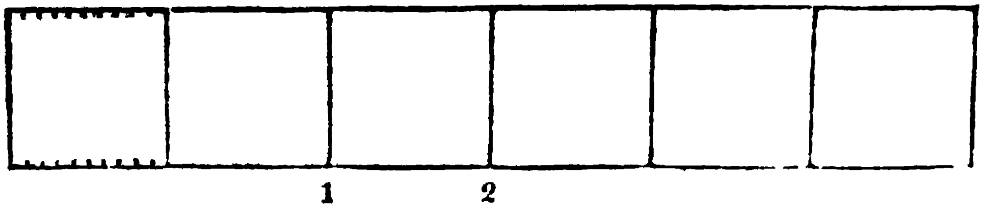
Under the right extremity of the first division place the figure 0, and under the remaining points of division place successively the figures 1, 2, 3, 4 and 5.



Draw a line above the former, parallel to it, at a distance equal to the length of one of the large divisions.

From the two extremities of the first drawn line, and from all the points upon it which mark the large divisions, raise perpendiculars meeting the above parallel. The upper and lower line will then both be divided into the same number of large divisions.

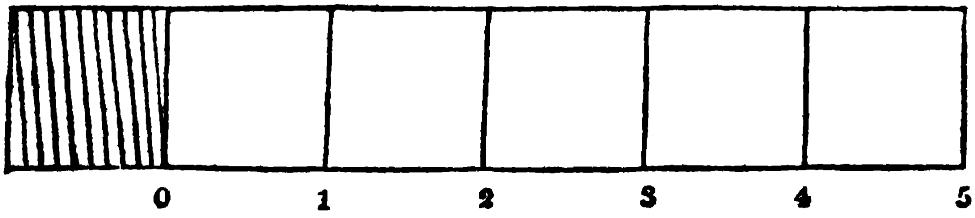
Subdivide the left division of the upper line or parallel also into ten equal parts; and the two lines will then be divided and subdivided exactly in the same manner.



From the left extremity of the first subdivision of the upper line, draw an oblique line to the right extremity of the first subdivision of the lower line.

From the left extremity of the second subdivision of the upper line, draw an oblique line to the right extremity of the second subdivision of the lower line.

And in like manner from the left extremity of every remaining subdivision of the upper line, draw an oblique line to the right extremity of its corresponding subdivision on the lower line.



Subdivide that perpendicular which connects the left extremities of your two first drawn lines into ten equal parts; and through the various points of subdivision thus found, draw right lines parallel to your original line.

Mark every second subdivision on your lower line from 0 towards the left extremity of the line by the numeral figures 2, 4, 6, and 8.

In like manner mark every second subdivision on your left perpendicular by the numeral figures 2, 4, 6, 8, commencing from the bottom upwards.

For the method of subdividing the perpendicular, drawing the parallels, and marking the subdivisions, as directed in the three last paragraphs, the Teacher will refer to the next figure, in which these particulars are explained.

Your problem is now performed: you have made a diagonal scale.

It is proper here to observe, that this scale like all the others which you have hitherto made, ought to have been divided into eleven equal parts, in order that the numeration might extend from 0, 1, 2, 3, &c. regularly as far as 10.

If you should ever wish to make a diagonal scale for use, you must therefore recollect to divide it in the above manner. My reason in making you now use fewer than the proper number of equal parts was in order to save you trouble, on account of the difficulty, which I know there is, in making many small subdivisions accurately upon a slate. But if you had been practising on paper, I should have made you in the first instance draw a line eleven inches long, and divide it into eleven equal parts; after which the remaining operations would have been exactly the same as those of your present example.

I shall now teach you the method of measuring distances by the diagonal scale.

Let us suppose that our present scale represents feet. It may of course also be applied to yards or to any other measure or dimension that the person who uses it thinks proper.

It is a rule in diagonal scales, that the subdivisions on the perpendicular, are each one tenth part of the subdivisions on the base, and that the subdivisions on the base are one tenth part of the divisions.

Consequently if the large divisions on your base or original line are units, that is to say single feet; the subdivision of the base will represent each one tenth part of a foot; and the subdivisions of the perpendicular will each represent one hundredth part of a foot.

If the large divisions on your base represent ten feet each, then the subdivisions of the base will represent each one foot; and the perpendicular subdivisions will represent each of them one tenth part of a foot.

Also if the divisions represent hundreds, the subdivisions of the base will represent tens; and the perpendicular subdivisions will represent units.

But if the divisions represent thousands; the subdivisions of the base will represent hundreds; and the perpendicular subdivision will represent tens.

This being explained, I shall first suppose, that the large divisions of my scale represent each one hundred feet; and that I am required to measure five hundred and forty-three feet, from my diagonal scale.

I first place one leg of my compasses at that point of the large divisions which is marked 5; and extend the other leg to that point of the subdivisions of the base which is marked 4.

Here the Teacher will exemplify.

I have now got a distance equal to five of the large divisions of the base, which as I said before represent hundreds, added to four of the subdivisions of the base which as I said before represent tens. Five hundreds and four tens are five hundred and forty. I now only want three of my required number.

That leg of my compasses which was placed on division 5, I move upwards on the perpendicular raised from thence, as far as the point where the said perpendicular is intersected, by that parallel which is drawn through the third perpendicular subdivision of the scale.

The other leg of my compasses which was placed on subdivision 4 of my base, I also move upwards as far as the same parallel; and finding that it does not reach the point where this parallel is intersected by the oblique line which is drawn from the fourth subdivision of the base: I extend it as far as the said point of intersection, keeping the other leg of my compasses fixed in its last position.

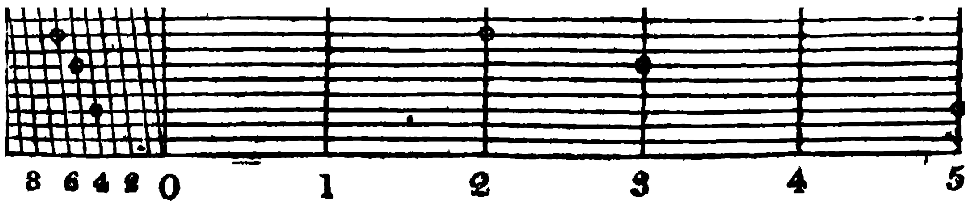
I have now got the exact distance of 543 feet, as was required.

I shall next show you how to take 356 feet from a diagonal scale.

I shall lastly show you how to take 268 feet from a diagonal scale.

The Teacher will exemplify as before. In the following figure, the points where the legs of the compasses ought to stand in the above three measurements, are marked by small circles, it being understood that the two legs must always be placed on the same parallel, in all measurements made upon a diagonal scale.

This figure would serve for the concluding operations of the problem in page 165, provided the small circles, which appear in it, were taken out.



Here the Teacher will exercise the learners in setting off distances, not exceeding 600 feet, by their diagonal scales, supposing, as in the above examples, that the divisions represent hundreds.

We shall now suppose that the large divisions of your scale represent thousands; consequently that the subdivisions of the base represent hundreds; and that the perpendicular subdivisions represent tens. The manner of measuring distances will be exactly the same as before.

I shall first show you how to measure 5430 feet.

I shall next show you how to measure 3560 feet.

Lastly I shall show you how to measure 2680 feet

The same points, which are marked with small circles upon the last figure, will also apply to these new measurements; and to all the following measurements relating to the diagonal scale.

Having exemplified the three measurements above mentioned, the Teacher will exercise the learners in similar operations, observing to choose distances not exceeding 6000 feet.

We shall now suppose, that the large divisions of your scale represent ten feet each: and consequently that the subdivisions represent feet, and tenths of a foot.

I shall first show you how to measure 54 feet and 3 tenths of a foot.

I shall next show you how to measure 35 feet and 6 tenths of a foot.

Lastly I shall show you how to measure 26 feet and 8 tenths of a foot.

Here the Teacher, after exemplifying, will exercise the learners *in similar operations, observing to choose distances not exceeding 6 feet.*

We shall now suppose, that the large divisions of your scale represent units or single feet; and consequently that the subdivisions represent tenths of a foot, and hundredth parts of a foot.

I shall first show you how to measure 5 feet, 4 tenths, and 3 hundredth parts of a foot.

I shall next show you how to measure 3 feet, 5 tenths, and 6 hundredth parts of a foot.

Lastly I shall show you how to measure 2 feet, 6 tenths, and 8 hundredth parts of a foot.

Here the Teacher, after exemplifying, will exercise the learners in similar operations, observing to choose distances not exceeding six feet.

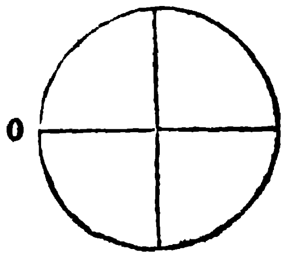
I shall next teach you the method of making scales for measuring angles.

It was before stated, that angles are always measured by degrees, or equal parts into which the circumference of a circle is supposed to be divided.

Write the words DEGREES OF A CIRCLE.

Describe a circle, and draw two diameters intersecting each other at right angles. These will divide your circle into four quadrants.

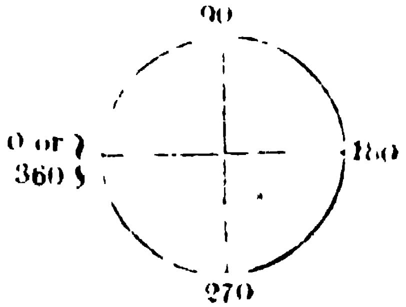
At the extremity of one of your diameters, mark the figure 0, to show the point from whence you mean to commence the divisions of your circumference.



A circle is always divided into 360 degrees. Your present circle is already divided into four equal parts by the intersecting diameters.

The fourth part of 360 being 90, at the extremity of the first quadrant from the point 0, you will mark 90: at the extremity of the second quadrant you will mark two nineties, which is 180; and at the extremity of the third quadrant you will mark three nineties, which is 270. At the extremity of the fourth quadrant, you will have made the complete round, which brings you back to the point 0, from whence you started. The whole circumference being equal to 360 degrees; this point may not only be marked 0, as being the commencement; but it may also be marked 360, as being the finishing point of the circumference.

Opposite to this point you will therefore write the word *or* and the number 360, in addition to the figure 0, which is already marked there.



In surveying instruments, such as theodolites, where the degrees are marked all round the circumference of a circle, the number 360 only is placed at the above point; the persons who use these instruments being supposed to know, that this also signifies the point 0.

A right angle is an angle of 90 degrees, as you may perceive by inspecting your present figure; and two right angles are equal to 180 degrees.

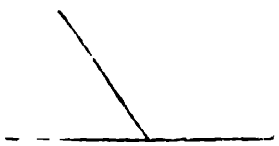
You cannot, properly speaking, use the expression an angle of 180 degrees; because if you suppose two radii of a circle to move upon the center as a pivot; when you extend them so far as to make them comprehend a part of the circumference equal to

180 degrees, they will then be exactly in the same right line, forming a diameter.

Consequently if any right line stands upon another, no matter whether perpendicularly or not; the two angles thus formed will be exactly equal to 180 degrees, or to two right angles, which is the same thing.

Also, if any two, three, or more right lines stand upon another right line, so as to meet and form angles, at any point in the latter line, on the same side of it; all the angles, thus formed, will be equal to 180 degrees or to two right angles.

For instance I shall first draw a right line; and secondly I shall draw another line standing upon it. The two angles thus formed are equal to two right angles.



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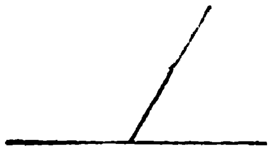
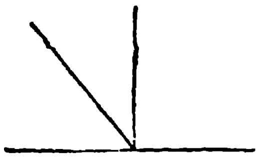


Now I shall draw some more lines, also standing upon the first drawn line, but all of them meeting in the same angular point.



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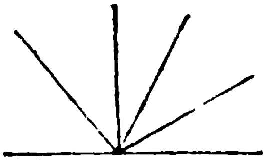


The three
five
or other number of
angles thus formed are exactly equal to two right angles, or to 180 degrees.

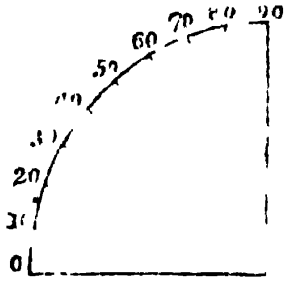
Here the Teacher will rub out the figures which he drew to illustrate the above remarks upon angles, that are equal to 180 *degrees, when added together.*

We shall now proceed with the divisions of the circle.

If you understand the manner of dividing a quadrant, you will not find any difficulty in dividing the whole circle. You will therefore rub out your circle, and draw a quadrant on a larger scale, placing the center from whence you describe the arc of the quadrant, nearly in the middle of your slates.

Divide the arc of your quadrant into nine equal parts, by trials with your compasses, each of which will be equal to ten degrees.

Place the figure 0 at one extremity of your quadrant, and from thence number the above divisions regularly with the numbers 0, 10, 20, 30, 40, &c. as far as 90.



The operations which you have just performed, being similar to many parts of your former problems, cannot be considered as forming a new problem, in themselves; they merely serve to explain the divisions of the circle. I shall now proceed to teach you, how these divisions may be applied to the measurement of angles.

This is done by forming them into a scale; which is called a scale of chords.

Write the words SCALE OF CHORDS.

A chord was before defined: it is a right line joining the two extremities of an arc.

The term chord of an arc may also signify the exact distance, or length, between the two extremities of an arc, although a right line may not actually be drawn in order to represent this distance.

This being explained, we shall proceed with our problem.

PROBLEM LIV.

TO MAKE A SCALE OF CHORDS.

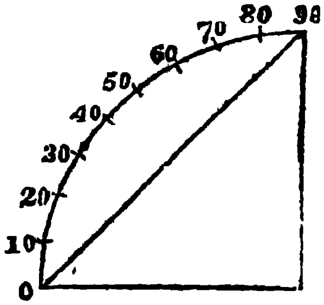
When the Teacher proceeds regularly, it will not be necessary for him to read the first of the following orders; the operations therein mentioned being already performed: he will therefore commence at the third paragraph, at the words " Draw the chord," &c.

But if in going through this course a second or third time, he should judge it most expedient to choose various problems separately, without attending to the regular order of the book; he must then, in performing this problem, commence as follows, and read every paragraph regularly until the whole of the operations are finished.

Draw a quadrant on as large a scale as your slate will conveniently permit; and divide the circumference of it into nine equal parts, each of which will represent ten degrees.

Place the figure 0 at one extremity of your quadrant, and from thence number your divisions regularly with the numbers 10, 20, 30, &c. as far as 90.

Draw the chord of your quadrant; or the chord of 90 degrees, which is the same thing. This is done by drawing a right line from the point 0 to the point 90.



This right line will be the total length of your scale of chords, which is not to go higher than the chord of 90 degrees.

The next operation necessary will be to mark upon it, the length of the chord of 10 degrees, also of the chords of 20 degrees, of 30 degrees, of 40 degrees, and in short of all the arcs into which your quadrant is divided.

The length of the chord of 10 degrees is equal to the distance from the point 0 to the point 10.

The length of the chord of 20 degrees is equal to the distance from the point 0 to the point 20.

The length of the chord of 30 degrees is equal to the distance from the point 0 to the point 30.

And in like manner, the lengths of the chords of 40, 50, 60, or any other number of degrees, are respectively equal to the distance, between the point 0 and the corresponding points, marked 40, 50, 60, &c. &c. on the arc of your quadrant.

The lengths of the chords required being therefore all measured from the point 0, the simplest way of transferring these various lengths to the chord of 90 degrees, is to place one leg of your compasses in the point 0; and from that point as a center, with radii extending as far as the points 10, 20, 30, 40, &c. &c. marked in the curve, to describe arcs meeting the chord of 90 degrees.

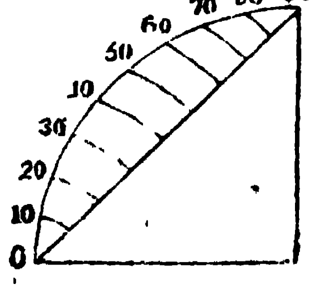
For instance from the point 0 as a center with a radius extending as far as the point 10, describe an arc meeting the chord of 90 degrees.

Here the Teacher will exemplify.

From the point 0 as a center, with a radius extending as far as the point 20, describe a second arc also meeting the chord of 90 degrees.

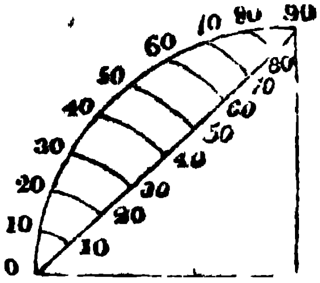
Here the Teacher will exemplify.

In like manner, invariably using the same point 0 as a center, but always choosing a new point of division on the curve for the extent of your radius, describe arcs meeting the chord of 90 degrees, until no more divisions remain.



The line which represents the chord of 90 degrees is now divided into the same number of parts, as the arc of your quadrant.

Number these new divisions in the same manner as the former, commencing at the point O, and marking under each remaining point the numbers 10, 20, 30, 40, &c. &c. in regular order as far as 90.



The line, thus divided and numbered, is a scale of chords, by which you may measure the chord of 10, 20, 30, 40, 50, 60, 70, 80 or 90 degrees; but you cannot yet measure any number of degrees, although under 90, which does not exactly agree with one of these numbers. Your scale is therefore incomplete until you have got it subdivided into parts of single degrees each.

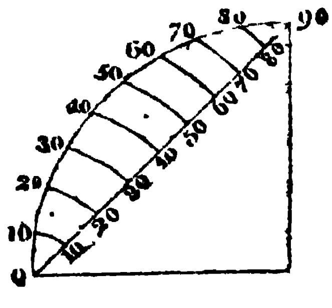
The process of subdividing your scale into single degrees is done exactly on the same principle, by which you have already divided it.

The method of doing this is sufficiently simple; and may be clearly explained to you, without giving you the trouble of marking the whole number of degrees or subdivisions, that are necessary in a scale of chords.

Instead of subdividing each of the present divisions of your quadrant into ten equal parts, or single degrees; you will, therefore, only bisect them.

Every large division represents 10 degrees; your present subdivisions will therefore represent parts of five degrees each.

From the point O, as a center, with radii extending as far as each of your points of subdivision, one after another, until no more remain; describe arcs meeting the chord of 90 degrees, which represents the required scale.



The Teacher need not make the learners dot their last described arcs. They are dotted in this figure only for the sake of clearness.

Your scale is now divided and subdivided, in the same manner as the arc of your quadrant; so that you can measure from it every 5th degree. For instance you can measure the chord of 25, 35, 45, or 55 degrees, &c. &c.

It must now be perfectly evident, that, by the same method, you might make a scale of chords capable of measuring every single degree.

Having subdivided your scales into as many parts as you were directed; the problem is performed.

REMARKS ON PROBLEM LIV.

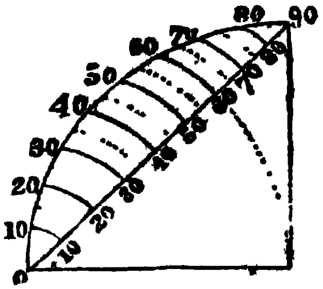
In your former scales, which you made according to the last problem, only one of the large divisions was subdivided; and yet you could measure any number of parts comprised within the length of the scale; whereas in a scale of chords, the whole of the large divisions must be subdivided before you can measure every number of single degrees necessary.

The reason of this is, that your former scales were all scales of equal or proportional parts; but a scale of chords is not a scale of equal or proportional parts.

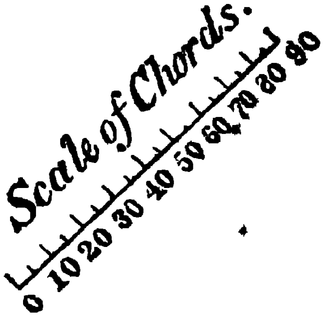
For instance, although the arc of 20 degrees is exactly twice as long as the arc of 10 degrees, measuring along the circumference of a circle; yet the chord of an arc of 20 degrees is not quite twice as long as the chord of an arc of 10 degrees: nor is the chord of an arc of 40 degrees twice as long as the chord of an arc of 20 degrees; nor are the chords of any arcs to each other in the same proportion as the numbers of degrees contained in their respective arcs. Consequently every degree or subdivision on a scale of chords being unequal, you cannot have a true scale of chords, without subdividing the whole of it into as many parts, as there are single degrees.

The same observation applies to all scales for measuring arcs or angles, of which there are several other kinds, which will afterwards be explained.

In a scale of chords, the chord of 60 degrees is always equal to the radius originally used, in describing the circle or quadrant, by means of which the scale was made.



Produce downwards the arc which was used in setting off the chord of 60 degrees, and you will see that this is the case: namely, that the chord of 60 degrees is exactly equal to your radius.



Rub out the whole of your figure excepting only your scale, over which you will write the words *scale of chords*.

PROBLEM LV.

TO MAKE OR MEASURE AN ANGLE OF ANY NUMBER OF DEGREES BY MEANS OF A SCALE OF CHORDS.

Let us first suppose, that the angle required to be made is an angle of $\left\{\begin{matrix}20\\35\\75\end{matrix}\right\}$ degrees.

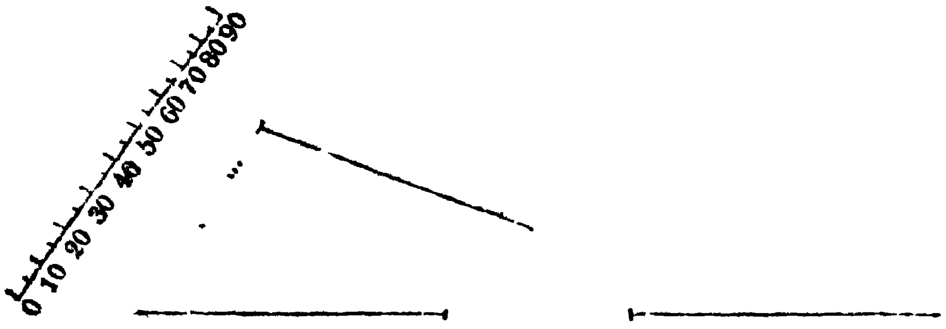
The Teacher will, of course, only choose one line or angle, at a time, and cause only one figure to be drawn; although in this and in the following examples, given in the book, several angles may be drawn together.

On some other part of your slates draw a right line, and mark a point in it, from whence the required angle is to be set off.

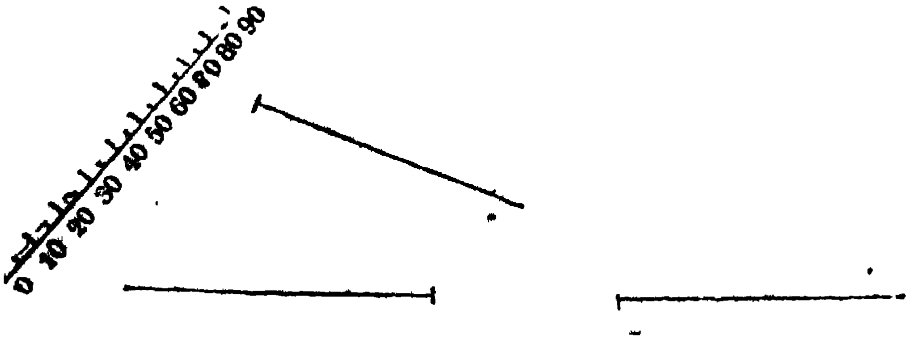
The scales of chords, constructed in the last problem, are supposed still to remain upon the slates of the learners: otherwise this problem could not be performed, unless they were provided with proper scales for the purpose.

In all figures, relating to such part of the following operations, as do not require actual measurements from the scale of chords, the scale is left out: but in figures which relate to actual measurements, a scale of chords, equal to that which appears at the end of the last problem, is inserted for the guidance of the Teacher.

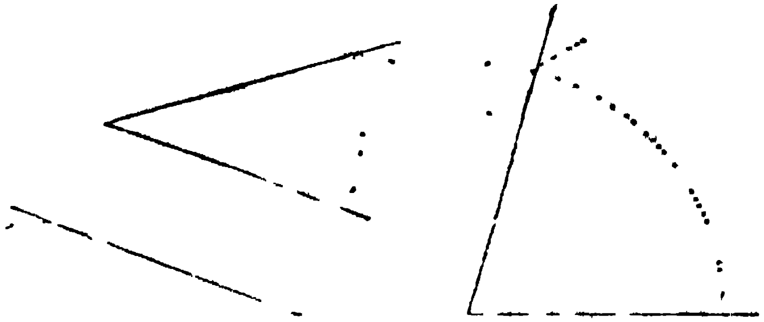
Take the length of the chord of 60 degrees from your scale, and with this distance as a radius, from the above point as a center, describe an arc on that side of the line, on which you propose to make your angle.



From the point where this arc meets or cuts the above line, as a center, with a radius equal to the chord of $\begin{Bmatrix}20\\45\\75\end{Bmatrix}$ degrees, describe a new arc intersecting the former.



From the proposed angular point, that is to say from the point first marked on your right line, draw a second line through the point of intersection of the two arcs.

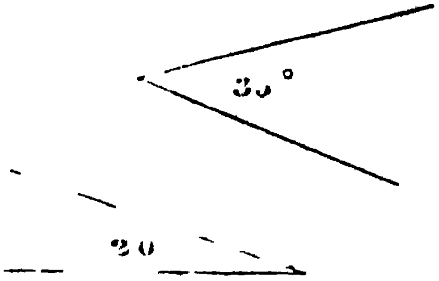


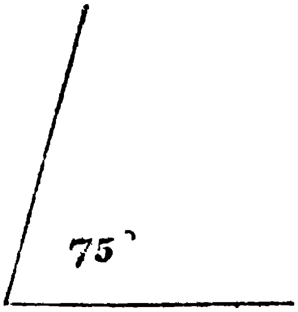
This last drawn line forms an angle with the former, which will be the angle required: consequently your problem is performed.

Rub out superfluous arcs.

It is often usual to mark opposite to an angle the number of degrees which it contains. The number is generally marked near the angular point. A small cipher, or figure of 0, placed over any number signifies degrees.

Mark, in the above manner, the number of degrees contained in the angle which you have just made.





Rub out your angle.

I shall now teach you how to measure any given angle by means of a scale of chords. We shall first suppose, that the given angle is an acute angle.

Draw an acute angle to represent the given angle.

From the angular point as a center, with a radius equal to the chord of 60 degrees, describe an arc for the purpose of measuring your angle.

Here the Teacher will exemplify.

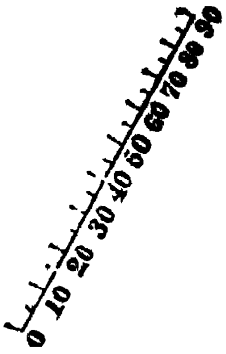
Take the length of the chord of the arc which you have just described in your compasses: that is to say, take the distance between the points, where your arc meets or intersects the two lines that form the angle.

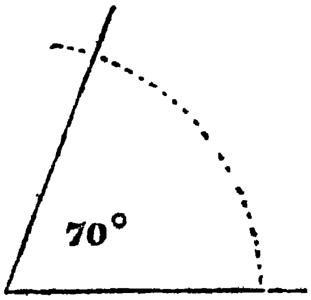
The next thing is to measure this distance upon your scale of chords.

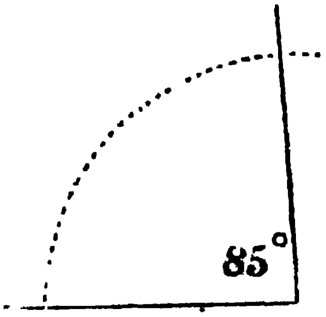
You see that as far as regards the arc which I have drawn upon the board: when I apply the chord of this arc to my scale of chords, it forms an angle of $\genfrac{}{}{0pt}{}{70}{85}$ degrees nearly.

The above are the numbers which suit the two figures here given, as an illustration of this operation: but as it is not likely nor necessary that the angle, which the Teacher draws upon the board, should correspond with either of these numbers; he will of course, in reading the above paragraph, make use of any other number, which he finds actually to agree with the measurement of his own angle.

I shall therefore mark my angle accordingly.







You will measure and mark your angles in the same manner: but if you find that they cannot be exactly measured by the subdivisions of your scale of chords; you must guess the odd number of degrees as near as you can.

Rub out your superfluous arcs.

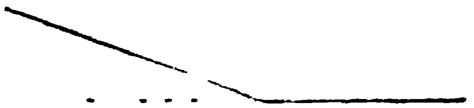
Rub out your figures.

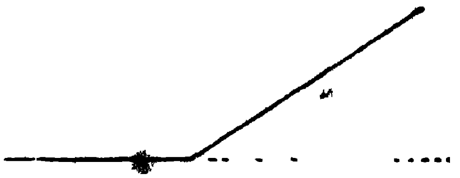
Obtuse angles, or angles of more than 90 degrees, may also be set off or measured by means of a scale of chords, by one measurement only.

I shall first teach you how to measure them.

Draw an obtuse angle to represent the given angle which is to be measured.

Produce one of the lines, which form your given angle, beyond the angular point; and dot this produced part.

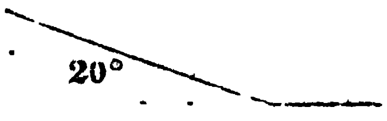


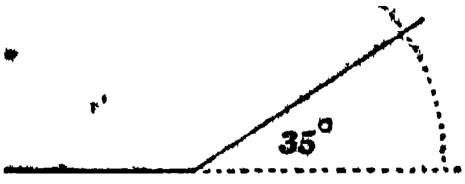


By so doing you have formed a new angle, which if added to your given angle will be exactly equal to 180 degrees, or to two right angles, as was before explained. This new angle will always be an acute angle, consequently you may measure it by your scale of chords.

Measure the new angle in the manner before explained, by first describing an arc with the chord of 60 degrees; and afterwards applying the length of the chord of this new arc to your scale of chords.

Following this method, the angle which I have drawn on the board, proves to be $\overset{20}{35}$ degrees. I shall therefore mark this number opposite to it.





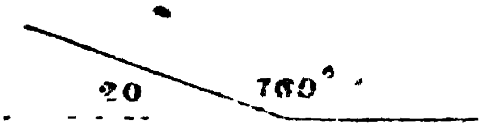
The Teacher of course, after measuring his angle, will not use the numbers 20 or 35, but any number which proves to be correct, these numbers merely applying to the two figures here given.

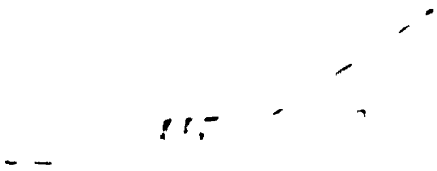
A scale of chords is not inserted in these figures, because angles of 20 and of 35 degrees were before set off according to a scale.

You will also mark your angles in the same manner.

The new angle of $\genfrac{}{}{0pt}{}{20}{35}$ degrees, which I have thus measured, if added to my given angle ought to make exactly 180 degrees.

I therefore subtract $\genfrac{}{}{0pt}{}{20}{35}$ degrees from 180, and $\genfrac{}{}{0pt}{}{160}{145}$ degrees remain: this is the exact measurement of my given angle. I shall mark it accordingly, rubbing out at the same time my superfluous arc.





You will in like manner subtract the number of degrees contained in each of your new angles from 180 degrees and the difference will be the exact value of your given angle, which you must write opposite to it.

When any arc is less than 180 degrees, the difference between the two is called the supplement of that arc.

In like manner when any angle is less than 180 degrees, the difference between the two is called the supplement of that angle.

For instance an arc or an angle of 130 degrees is the supplement to an arc or angle of 50 degrees: and in like manner, an arc or an angle of 50 degrees is the supplement to an arc or angle of 130 degrees.

Write the words SUPPLEMENT OF AN ARC.

I shall now teach you how to draw an obtuse angle, or an angle of more than 90 degrees, by means of your scale of chords, at one measurement.

Let us suppose that an angle of $\substack{160 \\ 145}$ degrees is required to be made.

Draw a right line to represent one of the lines of your required angle; after which it will be necessary to mark some point in it, to show the point from whence the angle is to be set off.

Although this point might be marked at any part of the line, the operation will be more clearly understood, if you take one of the extreme points of your lines for this purpose. You will therefore mark either extremity of your line accordingly.

Produce that extremity of your line where the point is marked, as far as you judge convenient; and let this produced part be dotted.

If the point had been marked near the middle of the line, this last operation would not have been necessary; as you will clearly understand when you proceed a little further.

At the point marked in your line, on the same side of the line proposed for your required obtuse angle of $\substack{160 \\ 145}$ degrees, but on the contrary side of the point, you must next make an acute angle equal to the supplement of your required angle.

The supplement of $\substack{160 \\ 145}$ degrees is $\substack{20 \\ 35}$ degrees.

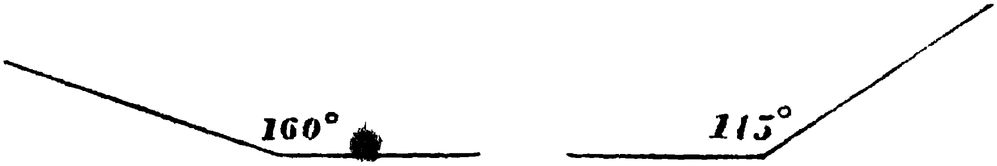
You will therefore, at the point marked in your line, make an angle of $\substack{20 \\ 35}$ degrees, in the manner directed, and number it accordingly.

A figure made to correspond with this operation, would be precisely the same as the second figure in page 181, *to which the Teacher may refer if necessary.*

The line, drawn in making this acute angle, forms also a second angle with the original line; which will be equal to $\substack{160\\145}$ degrees, as was required.

A figure, made to correspond with this operation, would be precisely the same as the figure in page 182, *to which the Teacher may refer if necessary.*

Rub out your superfluous angle, together with the dotted line, arcs, &c. used in describing it: and your operation is performed.



Rub out your figure.

REMARK.

In order to save trouble, a case of drawing instruments usually contains an ivory ruler, upon which are marked plane scales of various proportions, a diagonal scale, and one or two scales of chords for measuring angles.

There is also an instrument, called a protractor, by which angles may be set off or measured much more expeditiously and conveniently than can be done by a scale of chords.

Write the word PROTRACTOR.

PROBLEM LVI.

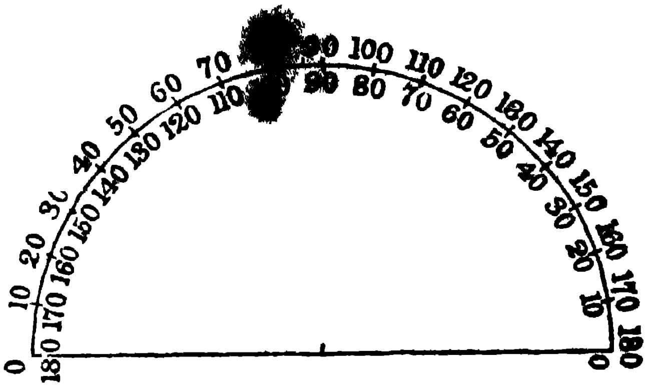
TO MAKE A PROTRACTOR.

Draw a semicircle: mark the center of it by a point; and divide the circumference into eighteen equal parts; each of which will be equal to 10 degrees.

You will therefore place the figure 0 under one extremity of your semicircle, from whence you will mark the divisions regularly with the numbers 10, 20, 30, 40, 50, &c. &c. as far as 180.

Here the Teacher will exemplify, and see that the learners perform the operations directed, before he makes them proceed further.

For the conveniency of setting off angles on different sides of the same line, without the trouble of calculating supplements, it is proper that the degrees of a protractor should be numbered in two contrary ways.



You must therefore put double numbers to all your divisions accordingly, commencing at the contrary extremity of the same semicircle, where you will place a new figure of 0, and from thence you will go regularly round the curve as before, but in a contrary direction, with the numbers 10, 20, 30, 40, &c. as far as 180.

I shall now teach you the method of setting off angles by means of a protractor.

One protractor ought to be provided for every five or six learners, but for the sake of economy, wooden protractors, divided in the manner shown in the above figure, may answer the purposes of instruction. The Teacher ought of course to have a larger wooden protractor to use upon the board.

We shall first set off an angle of $^{130}_{\ 50}$ degrees.

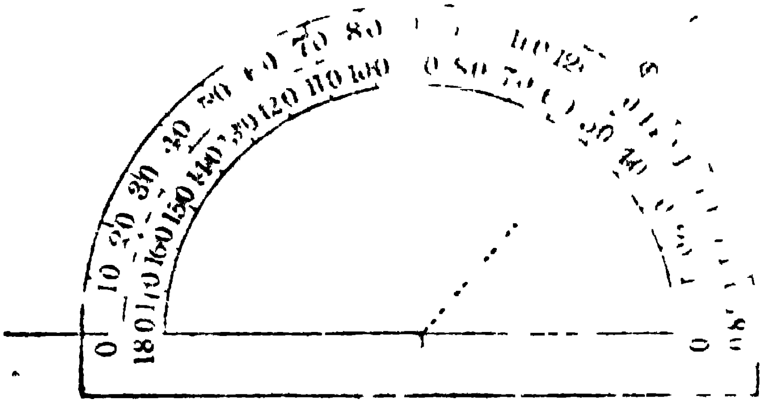
Draw a right line, and mark a point on it, to represent the point from whence the required angle is to be set off.

Place one of the straight sides or edges of your protractor upon the above line, in such a manner, that the center of your protractor shall exactly agree or coincide with the point, which you have just marked upon the line.

Then close to the semicircular or numbered side of your protractor, but on the outside of it, mark a point upon your slate opposite to that division, which stands for 130 degrees.

The following figure represents a real protractor applied to the above line, havin[illegible]e straight side and a semicircular side, but open in the middle.

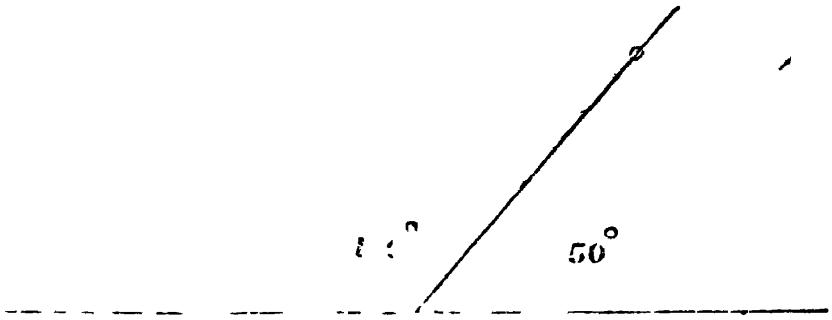
The dotted line is drawn merely as a guide to the Teacher, to show the direction of the point which ought to be marked, and the point itself is denoted by a small circle.



Remove your protractor altogether:

And from the point in your line, draw a right line through the last marked point.

This new line will form an angle of $\overset{130}{50}$ degrees with the former line, as was required. Mark your angle with the proper number of degrees, and your problem is therefore performed.



A protractor is also sometimes made in the form of a rectangle. For instance, in the flat ivory rulers or scales which generally form part of a case of drawing instruments, as was before mentioned, the degrees are always marked round three sides of the rectangle, which therefore correspond with the curved part of the semicircular protractor.

The remaining long side of the rectangle corresponds with the diameter of the semicircular protractor, and has a point marked in the middle of it, which in like manner corresponds with the center of the semicircle. This point is always placed in one of the long sides of the rectangle.

Rectangular protractors are not quite so convenient for practice as the semicircular ones, because they are much more liable to error, unless used with an uncommon degree of care and attention.

The reason of this is, that the divisions on the semicircular protractor are perpendicular to the semicircular edge of the instrument upon which they are marked; whereas in the rectangular protractor, some of them are very oblique to the edges or sides of the instrument.

The Teacher should have one or two rectangular protractors, to illustrate the above remark.

If you wished to make a rectangular protractor, it would be necessary first to draw a semicircular one upon the same center, dividing it regularly; after which the divisions may easily be transferred to the three proper sides of the rectangle, by merely drawing radii through the various degrees marked on the semicircle, and producing them as far as necessary: then the semicircle would be rubbed out.

Lastly, a double scale of degrees would be marked near the edge of the rectangle, on the three proper sides of it. This would render it complete as a rectangular protractor.

REMARKS.

When any line or instrument is marked with degrees, it is said to be graduated.

Write the words GRADUATED OR MARKED WITH DEGREES.

In your last problems, you have had frequent occasion to divide the circumference of a circle into degrees. There are various useful instruments divided in this manner.

A theodolite is an instrument of a circular form, upon which all the degrees of the circle are marked up to 360. It is used by land-surveyors in taking angles, as was before mentioned.

A quadrant is an instrument made in the form of a quarter of a circle, from whence it takes its name. This instrument is useful in measuring heights. It is also often used in gunnery, for laying guns and mortars at an elevation.

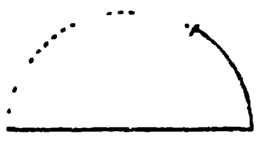
A sextant is an instrument made in the form of the sixth part of a circle. It is very useful in navigation, particularly for observing the sun's altitude or height above the horizon.

There is also a smaller kind of sextant, called a pocket sextant, which is useful in taking angles for small surveys.

Having made these remarks, I shall now proceed to explain to you some other kinds of scales, which are used for measuring arcs or angles.

DEF. 97. The sine of any arc is a perpendicular drawn from one extremity of it, upon a diameter passing through its other extremity.

Draw a semicircle: mark any point in the curve to divide it into two arcs; and dot one of the arcs thus formed. That arc, which is not dotted, will always be understood in the following observations, unless when the contrary is specified.



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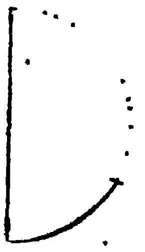
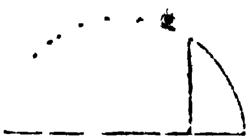


Draw a perpendicular to the diameter, from the point marked in the curve of your semicircle.



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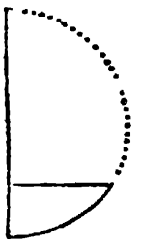


This perpendicular is the sine of your arc; it being drawn from one extremity of the said arc to a diameter passing through the other extremity of it.

Supposing that your arc contained any specified number of degrees, such as 60, then this perpendicular would also be called the sine of 60 degrees.

REMARK.

The same sine applies always to two arcs or angles, which are supplements to each other: for instance, the sine drawn in your present figure serves for both the arcs into which your semicircle is divided. It is the sine of the dotted arc as well as of the other; these two arcs being supplements to each other.

Thus the sine of 60 degrees is equal to the sine of 120 degrees: the sine of 40 degrees is equal to the sine of 140 degrees, and so on; these angles being supplements to each other.

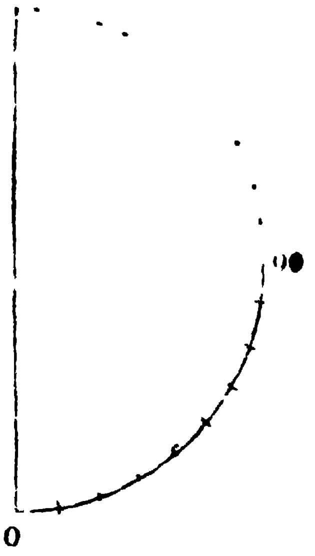
PROBLEM LVII.

TO MAKE A SCALE OF SINES.

Draw a right line directed towards the top and bottom of your slates: and upon this line describe a semicircle to the right of the line.

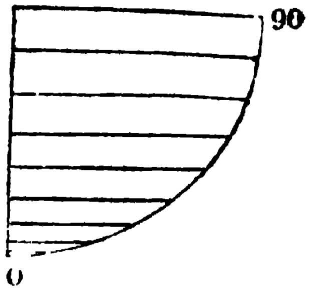
Bisect the arc of your semicircle, and divide the lower quadrant of it into nine equal parts, which will be equal to ten degrees each.

Place the figure 0 at the lower extremity, and the number 90 at the upper extremity, of the said quadrant.



From each point of division on the arc of your quadrant, draw a perpendicular to the diameter. These perpendiculars will be the sines of the various arcs marked on your quadrant: that is to say the shortest of them will be the sine of 10 degrees, the next will be the sine of 20 degrees, the next will be the sine of 30 degrees, and so on, as far as the sine of 90 degrees, which is equal to the radius of your semicircle; for it meets the center of it, as you may perceive.

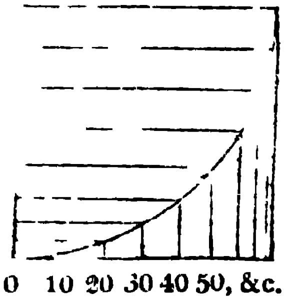
Rub out your upper quadrant which becomes superfluous.



Draw a right line from the point 0, parallel and equal to the sine of 90 degrees. This is to be your scale, upon which you must next transfer the lengths of the various perpendiculars, or sines, represented in your figure.

From all the points of division on the arc of your quadrant drop perpendiculars to the last drawn line.

Commencing from the point 0, mark the various points of division thus made upon your last drawn line, with the numbers 10, 20, 30, 40, 50, &c. &c. as far as 90.



If you pay attention to your figure, you may perceive that the divisions, thus made and numbered upon the above line, are equal to the sines of 10, 20, 30, 40 degrees, &c. Consequently this line forms a regular scale of sines; so that your problem is performed.

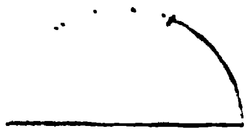
The original semicircle used in constructing your scale was placed in a particular position merely as being the most convenient: but the operations would, of course, have been equally correct, had it been placed in any other position.

Rub out your figures.

DEF. 98. The tangent of an arc is a right line drawn from one extremity of the arc, touching the circumference, and bounded by a radius produced through the other extremity of the said arc.

Write the words TANGENT OF AN ARC.

Describe a semicircle: divide it into two arcs by a point, and dot one of these arcs, of which we shall take no further notice.

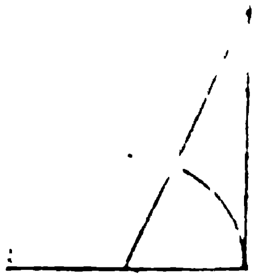


Draw a radius to the other extremity of your arc: I mean that arc which is not dotted.

From that extremity of the diameter, which bounds one side of the same arc, raise a perpendicular.

This perpendicular will touch the circumference.

Produce the perpendicular and the last drawn radius, until they meet each other and form an angle.



The perpendicular thus drawn is the tangent of your arc, being bounded on one side by a produced radius, and on the other by a diameter, both of which pass through extremities of the arc. But if the perpendicular were longer or shorter than the above proportion, it would not be a tangent to the arc which you have drawn, but to some larger or smaller arc.

Write the words TANGENT OF AN ARC.

The tangent of an arc takes its name from the number of degrees of a circle, which the arc contains; and may, for instance, be called the tangent of 20 degrees; or the tangent of 30 degrees, &c. &c.

DEF. 99. The secant of any arc is a right line drawn from the center through one extremity of the arc, and terminated by a tangent drawn from the other extremity of it.

Write the words SECANT OF AN ARC.

The produced radius of your present figure is the secant of the same arc, of which you drew the tangent.

Any line cutting a circle is a secant, as was before observed, but unless it were bounded in the manner above mentioned, it would not be the secant of your present arc.

Rub out your figures.

PROBLEM LVIII.

TO MAKE A SCALE OF TANGENTS.

Draw a quadrant of a circle, placing the center of it towards the bottom of your slates, and divide the curve into nine equal parts, each of which will be equal to 10 degrees.

Here the Teacher will exemplify.

From the outward extremity of the lower radius of your quadrant raise a perpendicular, which will be a tangent to the curve.

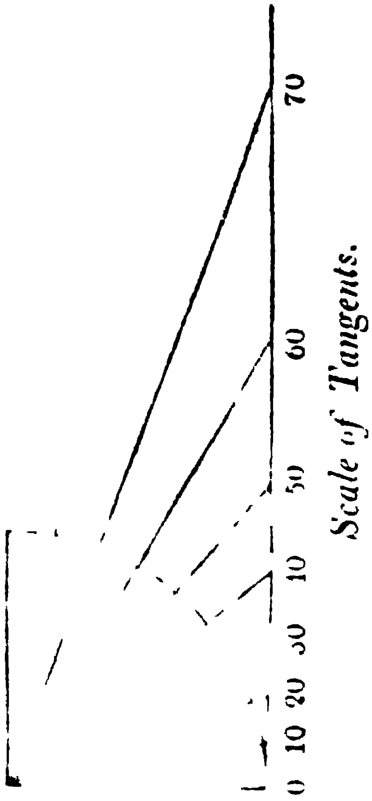
The Teacher will exemplify.

From the center of the circle draw secants through those seven divisions of the curve which are nearest to the said perpendicular; and produce them until they meet the perpendicular.

The Teacher will exemplify.

Place the figure 0 at the point from whence the perpendicular is raised, and from thence mark the remaining points into which it is divided by the numbers 10, 20, 30, 40, 50, 60, and 70.

The perpendicular thus divided and numbered is a scale of tangents. You will therefore write the words *Scale of Tangents* near it.



REMARKS ON PROBLEM LVIII.

There is no tangent of 90 degrees, because a secant drawn through the extremity of the arc of 90 degrees, or through the extremity of the quadrant, would be parallel to the perpendicular, which serves for your scale. Consequently they will not meet although produced ever so far.

In regular scales of tangents, which are drawn on the ivory rulers above mentioned, it is usual to carry the divisions and subdivisions no farther than 75 degrees: because to extend them above that proportion would require scales immoderately long.

The tangent of 45 degrees is equal to the radius of the quadrant: consequently when a slope of earth is set off at an angle of 45 degrees, the base of the slope is equal to its height. This is very common in field works.

Mortars are also generally laid at an angle of 45 degrees.

PROBLEM LIX.

TO MAKE A SCALE OF SECANTS.

A scale of secants cannot be made, without first going through all the operations that are necessary for drawing a scale of **tangents.**

In doing this problem separately, it will therefore be necessary for the Teacher to go back to the last problem, and cause the learners to perform the operations therein contained, after which he will proceed as follows.

On the scale of tangents, which you have made, you may observe, that the lines drawn through the several points of division marked on your quadrant, are the secants of the various arcs bounded by these points.

Consequently the only thing necessary is to transfer these secants to a regular scale.

Produce, beyond the curve of your quadrant, that radius of it which is parallel to your scale of tangents. The line thus formed is to be your required scale of secants.

The Teacher will exemplify.

From the center of your quadrant, as a center, with a radius equal to the secant of 10 degrees, describe an arc meeting the produced radius.

The Teacher will exemplify.

From the center of your quadrant, with a radius equal to the secant of 20 degrees, describe a second arc also meeting the produced radius.

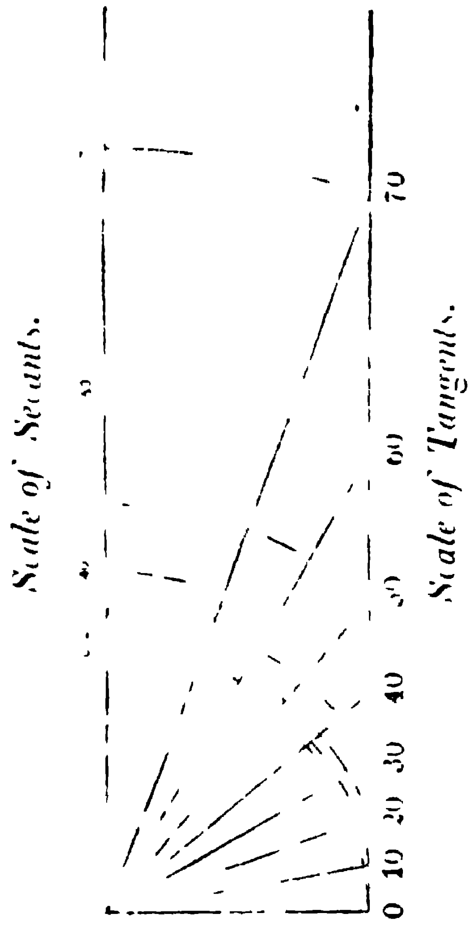
The Teacher will exemplify.

In like manner, using always the same center, with new radii respectively equal to the secants of 30, 40, 50, 60, and 70 degrees, describe arcs also meeting the produced radius.

The center of your quadrant is the commencement of your scale of secants, from whence all measurements should be taken; and your produced radius, now that it has been divided by the above arcs, is the required scale.

At the extremity of your original radius, that is to say at the point from whence it begins to be produced in order to form the scale, you will not place any number: and in a finished scale you would rub out that point, because it corresponds with the length of the secant of an arc of no degrees, which is not of any use.

The remaining points of division marked on your produced radius must be numbered regularly from thence, with the numbers 10 20, 30, 40, 50, 60, and 70.



Number them accordingly, and write near your scale the words *Scale of Secants*, in order to distinguish it from the Scale of Tangents which you have already drawn.

A scale of secants is seldom made longer than 75 degrees, for the same reason which applies to a scale of tangents; and the secant of 90 degrees, like the tangent of the same arc, is impossible, because if you produce it ever so far you will find no end to it; it being parallel to the tangent which it ought to meet.

In Geometry, when any line is in this predicament it is said to be of infinite length.

Write the words THE TANGENT OF 90 DEGREES IS OF INFINITE LENGTH.

Write also THE SECANT OF 90 DEGREES IS OF INFINITE LENGTH.

When scales of chords, sines, tangents, and secants, are all marked together upon the same ivory or wooden ruler; it is always understood that the quadrants used in constructing the whole of them must be described from the same radius; otherwise these scales would not be propotional to each other, and consequently could not be made use of in the same operation.

I before observed, that in a scale of chords the radius is equal to the chord of 60 degrees: in a scale of sines, it is equal to the sine of 90 degrees: in a scale of tangents, it is equal to the tangent of 45 degrees: but in a scale of secants, the radius is of no value; because it is shorter than the secant of the smallest possible arc that can be drawn.

A scale of secants is the least useful of any which you have yet drawn; because all the operations, in which it might be employed, can be performed without it, by means of scales of sines and tangents; and these last-mentioned scales are more convenient for practice.

If you have paid proper attention, so as to understand the various instructions which have been communicated to you, and the operations which have been performed; you have now learned enough of Practical Geometry, and may commence Plan-Drawing.

DIRECTIONS AND OBSERVATIONS

RESPECTING THE

BEST MODE OF INSTRUCTION.

SUCH directions, as were necessary for explaining the general principle, upon which the above Course of Practical Geometry ought to be conducted, have already been given.

Further and more detailed directions shall now be added, which are of essential importance, but could not have been introduced into the body of the Course, without creating confusion. These directions are calculated to obviate or remove the various difficulties which will be found to stand in the way of improvement: they are recommended in consequence of much observation and reflection: and they will apply not only to Practical Geometry, but also to the elementary parts of Fortification contained in the remainder of this Course, which may be taught exactly in the same manner, actual experience having fully proved the efficacy of this system of instruction, in both these branches of study.

In carrying on the Course, the learners should be placed in rows, upon forms or benches with narrow tables before them: and a stage or platform should be raised at one end of the room, for the Teacher to stand upon whilst he delivers his instructions; in order that the whole class may be able to see distinctly every thing which he writes or draws upon the board.

The Teacher must always give the word "ATTENTION," previously to reading any definition, order, or remark from the book; and ought to take particular care to see that this caution is obeyed. He should read every thing in a loud voice, and in a slow, clear, and distinct manner. It is also often proper to read the same definition, caution, or title of a problem, twice over.

When it is proposed to commence teaching a set of men totally uninstructed, they must be divided, either by sitting on distinct seats, or otherwise, into parties of not more than six or eight men each. In a very short time, three or four days, for instance, the Teacher will be able to distinguish the difference of talent in the learners; and will accordingly place those men who show most ability and attention at the head of their seat or class, who will from that time be called monitors. Until he ascertains who are the most fit for monitors, he must inspect the performance of every man himself. Afterwards he will inspect only the performances of these monitors. If he finds that they have understood his directions, and executed them correctly, he will order them each to inspect their seat, which will be done accordingly: but if any monitor should be wrong, he will go on inspecting the performance of the second, third, or fourth man of the same seat; and whoever is right must be immediately promoted to the head of the seat, in place of the former monitor.

In this manner, one Teacher may instruct thirty men at the same time, with great facility. For if we suppose these men to be placed in five seats or rows, or otherwise to be divided into five parties of six men each; then there will of course be five heads of seats or monitors. The Teacher will consequently have to examine the performances of five monitors; and each monitor will have to examine five learners.

But if the number of learners much exceeded thirty, for instance if there were sixty men employed at the same time;

then it would be necessary to have an assistant teacher in addition to the monitors. The principal Teacher, after giving his directions, and exemplifying them upon the board, would inspect the performances of five monitors: the assistant Teacher would in like manner inspect the performances of five other monitors; and these ten monitors would each inspect the performances of five learners. Out of sixty men, taken indiscriminately, although none of them may ever have had any previous instruction, you will always find some one of sufficient capacity to be able to act as assistant Teacher, after a few days' trial.

Sixty men may conveniently be instructed at the same board, if a room sufficiently large can be had for the purpose; but if the number of learners much exceeded sixty, it would be best to divide them, and place them in separate rooms, under different teachers.*

It was before observed, that the Teachers must take particular care, that the ablest man of each class or seat is placed at the head; and that every time the head of the seat commits an error, he must be made to resign his situation of monitor to the next in succession who happens to be right: In like manner, the heads of seats or monitors, in examining the performances of the men under their charge, are to see that every man who is correct in his operations, shall sit above those who make any error. The learners are also to be directed to claim their right and appeal to the Teacher, in case the monitor or head of their seat has neglected to promote them to a higher place, when entitled to it. In short, too much attention cannot be paid to this most essential point; without which there can be no emulation, slovenly habits will creep in, and little improvement ensue.

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* A class of sixty men requires a very large room. Necessity would therefore in most cases oblige a greater number to be divided, even if it were not more convenient in other respects.

When the Teacher wishes to examine what has been done, if the operation is of such a nature, that he can judge of its accuracy by inspection, he will give the word "SHOW SLATES;" upon which the learners will hold up their slates in such a position that a person passing in front of them between the rows of seats, may be able to inspect their respective performances with ease: consequently every learner must have the back of his slate turned towards himself. After any man's slate has been inspected, he will put it down on the table in its proper position, without waiting for the rest of the class.

This method of inspection will apply not only to all the words and phrases, which the learners may be directed to write, but also to some of the simple figures used for illustrating the definitions, and even to some of the steps of the problems.

When the operation is of such a nature, that the accuracy of it cannot be known by mere inspection, but requires measurements with the compasses, or the use of the triangle and ruler, the Teacher will then give the word "PROVE FIGURES;" upon which each learner will be ready with his instruments to show the Teacher in what manner he performed the last directed operation, or drew the last step of the figure, which appears on his slate.

The Teacher might also prove the accuracy of the performances of the learners, by taking up their instruments in passing, and therewith examining the respective performances himself: but the former method is much better, because it often happens, that a man may have performed his operations accurately, whilst at the same time he used his instruments in some awkward, clumsy, or inconvenient manner. This will be a great bar to further improvement, which the Teacher will not be able to remove, unless he makes the learners prove their own figures themselves in his presence.

As soon as the learners have performed any operation required, they are to lay down their instruments, and sit upright without touching the table, as a signal that they have finished. When the Teacher sees that the whole have finished, he will either give the word "SHOW SLATES," or "PROVE FIGURES," or such other word of command as he may judge proper.

The learners, who have performed any operation, are never to move their slates, except when the word "SHOW SLATES" is given. After all the other words of command, they will sit attentively waiting to be examined.

It is a general rule in respect to the problems, as has been stated in various parts of the Course, that each of them should at first be performed two or three times over, before the Teacher allows the learners to proceed further. Sometimes it may even be proper to repeat a problem oftener than three times. After the problem has been performed once or twice, the Teacher may, if he thinks proper, merely read the various directions necessary, and cause the learners to execute them step by step, without drawing any figure on the board for their guidance.

When he follows this method, the Teacher must take care to caution the learners that they are not to perform any operation of the problem, although they may perfectly understand the whole of them, until the direction for performing that particular operation shall have been read. This is necessary, to prevent the learners from working mechanically, without attending to the directions of the Teacher; which they will always be apt to do if left to themselves. In order to inforce proper obedience to this rule, every man, who oversteps the directions of the Teacher, must lose his place, although he may have been going on perfectly right in other respects.

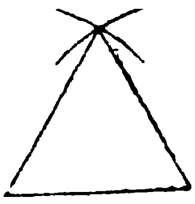
After a problem has been two or three times repeated in the usual form, and the slates cleaned for another repetition, the

Teacher may simply give the following word of command, "PERFORM THE SAME PROBLEM AGAIN." In this case he will neither read any of the directions, nor draw any figure upon the board for their guidance, but will leave the learners to perform the whole of the operations necessary, according to the best of their recollection. This method is a good trial of ability and attention, and should therefore be used as often as the Teacher finds it expedient, particularly towards the end of the Course, when the learners ought to be able to comprehend every thing with facility.

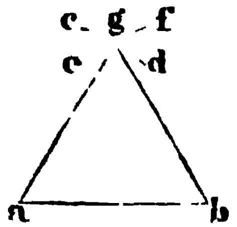
At the end of a problem, whether performed step by step or otherwise, the Teacher should give the word "EXPLAIN METHODS." Then, in going round to inspect the performances, he will cause each of the monitors to give an account in what manner he performed the various operations which were necessary to complete the figure that appears upon his slate; and the monitors, in like manner, after being thus examined, will require a similar explanation from each of the learners under their respective charge.

In explaining the method, according to which they performed any problem, it is necessary that the learners, when they say that they used any particular point as a center, or that they described any arc, or drew any line, should always, as they go on, point out to the Teacher or monitor, who examines them, the particular point, arc, or line, to which they allude.

Let us suppose, for instance, that problem 6, which relates to the making of an equilateral triangle upon a given right line, has just been performed; the following figure will then appear upon the slates of the learners.



Before we proceed further, the various points, arcs, and lines on the above figure, shall now be marked with small letters of the alphabet; in order that this mode of examining the learners may be more clearly understood: but it is to be observed, that in



real practice no letters would be used. In the present instance, they are only intended for the information of such readers as may not have had an opportunity of seeing the Course actually carried on.

This being premised, we shall now suppose that the word "EXPLAIN METHODS" is given. Each learner will then give an account of his operations in the following manner.

Here the learner is supposed to speak.

" I first drew this line *(pointing to the line marked* a b *in the*
" *figure)* to represent the given right line. I took the length of
" it in my compasses, and from this extremity of it *(pointing*
" *out the point* a) as a center, I described this arc *(pointing to*
" *the arc* c d). I then took this other extremity of my given line
" *(pointing out the point* b) as a center, and with the same radius
" as before, I described this second arc *(pointing to the arc* e f).
" From the two extremities of my given right line, I then drew
" these two lines *(pointing out the lines* a g *and* b g) to the point
" of intersection of my two arcs *(pointing to the point* g).
" This completed the equilateral triangle which I was directed to
" make."

This may serve as a specimen of the process of explaining methods. Some men will of course be able to explain themselves more clearly, and in better language than others. If the Teacher sees that they thoroughly understand what they have been doing, he will not be too particular in objecting to the manner in which the learners explain their methods, provided they make use of proper geometrical terms. But whenever they make use of such terms as striking a line, instead of *drawing a line;* striking or sweeping an arc, instead of *describing an arc;* or when they talk of two lines being square to each other, instead of being *perpendicular;* he must always be sure to correct them. Indeed, one principal use of the practice here recommended, is to oblige the learners to pay proper attention to the geometrical terms.

For the same reason, the Teacher either before or after he requires a learner to explain the method, in which the latter has performed any problem, may sometimes put the question, " What have you done?" The answer which the learner ought to make should correspond nearly with the title or heading of the problem.

For instance, after performing the sixth problem, a learner might say, " Upon this given line *(pointing to the line* a b) I " have described this equilateral triangle *(pointing to the triangle* " a b g)."

As there are more methods than one of performing some of the problems, the learners in their answers to the above question should also state what method they followed, in respect to these particular problems.

For instance, after performing problem 1, which relates to the drawing of parallels; a learner might either reply, " Through " this given point, I have drawn this line parallel to this given " right line by a ruler and compasses:" or if he used the second method of performing the same problem, he might say, " Through this given point, I have drawn this line, parallel to " this given right line, by a triangle and ruler."

The word " PROVE FIGURES" should never be used at the end of a problem, this word of command being only applicable to the simple figures which are drawn for the purpose of illustrating the definitions, or to the preliminary operations in a problem, when performed and examined, step by step. But it is to be understood that even when the word " EXPLAIN METHODS" is given, the learners should in general not only explain their operations in words, but also prove the accuracy of them, as they go on, by their instruments. If, however, the Teacher, after long trial, finds that any of the learners are always accurate in their operations, the proving may be dispensed with, when he examines these individuals.

It may often happen, even when the word "SHOW SLATES" is proper; that the Teacher, or monitors, in inspecting the performances of the learners, may have doubts as to the accuracy of some particular individual: in that case they will require him to prove his figure.

For instance, if we suppose that the order were given to draw an acute or an obtuse angle. In general the accuracy of either of these operations may easily be known by inspection: but if through carelessness, or from not having well understood the definition, any of the learners should happen to draw angles nearly resembling right angles; then it would, of course, be necessary to make these individuals prove their figures.

It is not advisable to go through the whole course of Practical Geometry at once. The Teacher should therefore divide it into a certain number of portions, such as he judges convenient. After going through each portion, he must examine the learners in respect to what they have done, not questioning them upon the various definitions and problems, in regular order, as they stood in the Course, but selecting them at random.

If, at any of these examinations, he finds the learners imperfect, he will cause them to go through the same portion of the Course a second and even a third time; examining them each time, before he allows them to proceed farther.

When the Teacher conceives that the learners are perfect in one portion of the course, he will make them begin a second portion of the same, but still adhering to the above system: and thus he will go on, taking one portion after another, always having recourse to repetitions and examinations, until the whole is finished.

The parts into which the Course of Practical Geometry may conveniently be divided are as follows.

DIVISIONS OF THE COURSE.

1st. From the beginning of the Course to Definition 70, inclusive (ending at page 44).

2d. From the commencement of the Explanation of the nature of some of the principal Solids (in page 44), to the end of Problem 33 (in page 71).

3d. From Problem 34 (in page 71), to the end of Problem 48 (in page 100).

4th. From Definition 87 (in page 100), to the end of the Remarks on the supplementary Problems (in page 149).

5th. From the commencement of Problem 52 (in page 149), to the end of the Course.

After each of these portions or divisions of the Course, the learners must be examined, as was before observed.

As far as regards the definitions, the examinations must be conducted in three different ways.

EXAMINATIONS IN RESPECT TO THE DEFINITIONS.

METHOD 1. The Teacher will name any term of Geometry and make the learners define it, by putting questions to them; for example:

Question. What is a solid?
Answer. A solid is that which has length, breadth, and thickness. *See Def.* 1.

Question. What are the boundaries of a solid?
Answer. Superficies.

Question. What is a superficies?

Answer. A superficies is that which has length and breadth only, but is supposed to have no thickness, it being merely the boundary or outside of a solid. *See Def.* 2.

Question. What is a plane superficies?

Answer. A plane superficies is that which is perfectly even; so that if you lay the edge of a ruler upon it in any direction, the ruler will touch it in every point. *See Def.* 3.

Question. What is a curved superficies?

Answer. A curved superficies means a crooked or uneven superficies; and is such as will not agree with the edge of a straight ruler laid upon it in any direction, &c. *See Def.* 4.

Question. What is a line?

Answer. A line is the boundary of a superficies. It is that which has length only, but is supposed to have no breadth nor thickness. *See Def.* 5.

In like manner, every definition in the Course may be made the subject of a question, which the Teacher will put to the learners. In their answers, it will be sufficient if they give the correct sense of the definition required, although they may not use the same words which are found in the book.

For instance, if we suppose the question to be put—"What is a right-angled triangle?" *See Def.* 30. The answer according to that definition ought to be as follows:

"A right-angled triangle, is a triangle which has one right angle."

But the reply would be equally correct if, in answering this question, a learner were to say:

"A right-angled triangle is a triangle which has two of its sides perpendicular to each other."

There will be many instances in which two answers equally appropriate may be made to the same question. The Teacher will therefore keep this in mind, and not object to any answer merely because it is couched in different words from those used in the book.

Enough has now been said upon this mode of examination, which is scarcely liable to error.

METHOD 2. The second mode of examination in respect to the definitions is exactly the reverse of the former. It consists in drawing, describing, or exhibiting some line, figure, or body, and asking the learners the geometrical term or name of it.

For example. The Teacher may sketch out figures on the board of various kinds, one at a time; and according to the nature of his figures he may put the following several questions.

Question. Supposing that these two sides of the triangle, which I have just drawn, are perpendicular to each other, what kind of triangle do you call it?

Answer. A right-angled triangle. *See Def.* 30.

Question. Supposing that its three sides are all of different or unequal lengths, what name do you give it in consideration of this circumstance?

Answer. A scalene triangle. *See Def.* 28.

Question. Supposing that the three sides of the new triangle, which I have just drawn, were all equal to each other, what name would you give it?

Answer. An equilateral triangle. *See Def.* 22.

Question. What name would you give it, in consideration of the nature of its angles?

Answer. An acute-angled triangle. *See Def.* 29.

Question. Supposing that the triangle, which I have just drawn, had one obtuse angle, what name would you give it?

Answer. An obtuse-angled triangle. *See Def.* 32.

Question. Supposing that two sides of it are equal to each other, what name would you give it in consideration of this circumstance?

Answer. An isosceles triangle. *See Def.* 27.

Question. Supposing that the quadrilateral figure, which I have just drawn, has all its four sides equal and its four angles all right angles, what name do you give it?

Answer. A square. *See Def.* 36 *and Problem* 9.

Question. Supposing that the angles of the quadrilateral figure, which I have just drawn, are all right angles, but that its sides are not all equal; what name do you give it?

Answer. A rectangle. *See Def.* 35.

Question. Supposing that in the quadrilateral figure, which I have just drawn, every two opposite pair of sides are parallel to each other *(here the angles are supposed to be oblique)*, what name do you give it?

Answer. A parallelogram. *See Def.* 34

Question. The right-lined figure, which I have just drawn, has six sides: supposing that these sides are all equal to each other, what name do you give it?

Answer. A regular hexagon. *See Deſs.* 44 *and* 52.

Question. You see the piece of wood which I have now in my hand. It has six faces or sides; each of which is a square; and all of which are equal. What name do you give it?

Answer. A cube. *See Def.* 73.

Question. Observe this side of it. If I apply a ruler to it in any direction, it always agrees with the ruler in every part. What name do you give it?

Answer. A plane superficies. *See Def.* 3.

Question. Observe the side of this other piece of wood. It will not agree with the ruler when applied to it in various directions. What name do you give it?

Answer. A curved superficies. *See Def.* 4.

In the two methods above mentioned, the Teacher must put questions to individuals; for instance, to one of the monitors, or to the second, third, or fourth man of a seat. If they cannot answer, he will call upon some other man of the same seat. And if no one of that seat can answer, he will call upon any man of the whole class.

In the following methods, the mode of examination is not carried on individually, but generally.

METHOD 3. The Teacher will give out the name of any geometrical line or figure, and order the learners to draw it. For instance, he may issue such orders as the following.

Draw a curved line. *See Def.* 8.
Draw a mixed line. *See Def.* 9.
Draw two parallel right lines. *See Def.* 10.
Draw two parallel curved lines. *See also Def.* 10
Draw a curve and a tangent to it. *See Def.* 11.
Draw an acute angle. *See Def.* 17.
Draw a right angle. *See Def.* 13.
Draw an obtuse angle. *See Def.* 18.
Draw a scalene triangle. *See Def.* 28.
Draw an equilateral triangle. *See Def.* 22.
Draw an isosceles triangle. *See Def.* 27.

Draw an acute-angled triangle. *See Def.* 29.
Draw a right-angled triangle. *See Def.* 30.
Draw an obtuse-angled triangle. *See Def.* 32.
Draw two or three parallelograms. *See Def.* 34.
Draw a square. *See Def.* 36 *and Prob.* 9.
Draw a rectangle. *See Def.* 35.
Draw a trapezium. *See Def.* 39.
Draw an irregular pentagon. *See Def.* 43.
Draw a regular hexagon. *See Def.* 44 *and* 52.
Draw an irregular decagon. *See Def.* 48.
Draw an arc and the chord of it. *See Def.* 58 *and* 59.
Draw a sector. *See Def.* 63.

After issuing any order of the above nature, the Teacher will either give the word "SHOW SLATES," or "PROVE FIGURES," as may be most expedient; after which the performances of the learners will be examined in the usual manner.

It now only remains to explain the mode of examination which should be used in respect to the Problems, which is sufficiently simple. Indeed, the nature of it might be inferred from what has already been said.

METHOD OF EXAMINATION IN RESPECT TO THE PROBLEMS.

When the learners are under examination in some portion of the Course, the Teacher will select any problem therein contained, and after reading the title or heading of it, and the particular method which is to be used, if there are more methods than one, he will order the learners to perform it. After giving this order, he is not to afford them any assistance by reading further directions, or by exemplifying upon the board.

When he sees that they have finished, he will give the word

"EXPLAIN METHODS;" and see that the performances are duly examined, in the manner before directed.

When this is done, he will proceed to a second, a third Problem, &c. always selecting them from the same portion of the Course, which is the subject of examination; but not in regular order as they stand in the book.

When the learners are under examination, the Teacher must always intermix the problems and definitions. In stating the above methods they were divided only for the sake of clearness.

After two or more portions of the Course have been performed, the Teacher may also, if he thinks proper, examine in respect to the whole of what has been done, without confining himself to those particular portions which were last performed.

The practice of causing the learners to explain their methods, and the examinations herein recommended, are of peculiar importance.

If the Teacher were to go on, according to the method which is most proper at the commencement of the Course, reading and exemplifying every thing, without requiring explanations, and examining the learners from time to time; although it would be impossible for any of them to be idle, as far as regards the practical operations; yet some of them would, in all probability, content themselves with copying the figures drawn upon the board, step by step; without endeavouring to understand the whole chain of operations in the problems they performed, and without attempting to remember, or perhaps even without listening to the various definitions and remarks read by the Teacher.

The consequence of this is, that at the first examination, those who have not paid proper attention are sure to expose their ignorance in an unpleasant manner; whilst the more diligent men distinguish themselves by the ability and readiness of their answers and the accuracy of their operations. In order to avoid a similar mortification, every man who has failed once, will exert himself to the utmost, before a new examination takes place.

Although a body of men may have commenced at the same time, it may be proper after a certain period to divide them into two classes, placing those who are most perfect in the first class, and allowing them to go on regularly with some new portion of the Course, whilst the others may be made to do the whole, or some part of what they have already performed, over again.

Whenever a body of men are under instruction; if they are allowed to change their seats at will, or to sit in the same order in which they placed themselves at the commencement of the Course, it will often happen, that four or five men of very superior abilities may all be seated together, whilst in some other seat most part of the men may be below par in point of capacity. If they remain in this confused state, it will be difficult or impossible to judge truly of the comparative ability of the learners, which, in the Royal Engineer department, it is very important to ascertain. The Teacher must therefore endeavour to put all the seats on a par, by dividing the men of abilities as equally as possible amongst them. This may easily be done in the course of a few days, or at least after the first examinations; and is an object which should be kept in view throughout the whole Course. After any arrangement has once been made for this purpose, the Teacher should not allow the men to change seats without his order or permission.

If these remarks are duly weighed, and the above rules carefully attended to, in conducting a Course of Instruction of this kind, it will be found, that the greatest attention and emulation will be excited on the part of the learners; and a rapid and general improvement will take place.

The Course of Instruction, herein laid down for the non-commissioned officers and soldiers of the Royal Engineer department, although the principle of it has been sanctioned by the highest authorities in that branch of his Majesty's military service, has not yet been carried into effect a sufficient length of time to be fully known through the whole corps; nor can the advantages of it yet be generally felt. It may, therefore, not appear superfluous, if some further observations are added, to explain the reasons, which render some course of instruction of this kind peculiarly useful and necessary; and which may be pleaded in favour of the particular system that has actually been adopted.

GENERAL OBSERVATIONS.

Every person who has paid attention to the mode in which works are carried on, in civil life, knows that the overseers and foremen of the various branches, who are employed in superintending the executive part, generally have some knowledge of Practical Geometry, and understand the nature of plans, sections, and models. At the same time, officers of engineers, and others who have had an opportunity of judging, will allow, that artificers so qualified in point of knowledge, are seldom to be found in the army.

But in garrisons at home and abroad, there are generally a proportion of ingenious and well-informed civil overseers and foremen, besides a number of skilful workmen aspiring to the same situations; who are either in permanent pay under government, or whose services might be called upon at a moment's warning. By means of these men, added to the military artificers, who either belong to the Royal Engineer department, or are usually attached to it, from amongst the troops in garrison, a commanding engineer finds no difficulty in carrying on any works of fortification, however extensive: and although he cannot avoid observing the comparative ignorance of the military artificers, it must be evident, that in such situations, an officer is not likely to feel any immediate sense of the necessity of endeavouring to instruct them.

When an army takes the field against an enemy, the case is widely different. There the engineers find themselves totally deprived of the assistance of the civil artificers by whose skill and ingenuity they were able to carry on their garrison duties, with ease to themselves, and advantage to the service:

Consequently, in executing their arduous duties in actual warfare, the officers of engineers have scarcely any resource but their own individual exertions, and the assistance of the non-commissioned officers and soldiers under their immediate command, whose want of knowledge and experience may then be deplored, but cannot be remedied.

It is true that military artificers, drawn from the battalions of the line, are occasionally put under the orders of the officers of engineers in the field as well as in garrisons; but these men are always much less efficient than those who actually belong to the Royal Engineer department; because if they were even more skilful and better instructed, which is not the case, they require to be so often changed in consequence of the exigencies of their regimental duties; and can so seldom be spared without prejudice to the efficiency of the respective corps to which they belong; that their services in the field, comparatively speaking, are of little value.

Here it will be proper, before we proceed further, to investigate the causes of that superiority of skill, which the civil artificer generally possesses over the soldier who has been brought up to the same trade. This question is of the utmost importance. Unless the root of an evil is truly appreciated, all attempts to remove it must be liable to failure.

The skilful and well-informed overseers and foremen of civil artificers, before alluded to, consist of two distinct classes of men. Some of them are young men, whose parents being in easy or even in affluent circumstances, have given them a good education, besides a trade, in order to qualify them for afterwards setting up in some business as master mechanics; where they have before them the prospect of acquiring a competency, or even of making a fortune. Such men, it may easily be conceived, will never willingly enlist as private soldiers.

The second class of skilful civil artificers, above alluded to, consists of men, who, though originally brought up in narrow circumstances, yet, by dint of ability added to keen observation and indefatigable perseverance, have contrived to educate themselves, with a view of rising to a higher situation. Before men, starting under such disadvantages, can acquire any considerable mechanical knowledge, they must pass so many years in the same habits, that they become wedded to their present prospects, and totally averse to a change of profession.

Considering these circumstances, it may appear, that even if the pay and allowances of the army were doubled or trebled, it is by no means to be expected, that any great number of civil artificers of approved knowledge and skill would thereby be induced to enter his Majesty's service; and it will not be a matter of surprise, that, as is actually the case at present, a man of this description seldom enlists, unless he has previously abandoned himself to vice and debauchery, in which case he will be an useless burthen to the service, at least, as far as regards the duties of the Royal Engineer department.

The artificers who enter his Majesty's service, are therefore in general very imperfectly instructed. Few of them understand more than the first common rules of arithmetic; and a considerable proportion of them are totally uneducated. As they enlist young, they seldom even have much practical skill in their respective trades.

The manual dexterity, in which they are deficient, is, however, often acquired by dint of long practice, in some particular employment; but they seldom or never endeavour to cultivate their abilities and improve their minds.

The reason of this indifference to improvement, on the part of the soldier, will be sufficiently obvious on a little reflection, and does not apply to the private only.

The military man of every rank, whose life is unsettled and uncertain, and whose subsistence is fixed, has not the same stimulus to mental exertion, as the civilian; who may either rise to comfort, and affluence, or may involve himself and his family in poverty and distress; in proportion as he cultivates, or neglects his abilities.

Superficial observers are apt to suppose that the comparative ignorance which may often be remarked on the part of the soldier, is owing to want of capacity. But it may be laid down as a maxim, that in any body of men, however indiscriminately collected together, there is always a latent fund of superior talent, which, if proper steps are taken, may be called into action for the benefit of the state: and although military men have less stimulus to individual improvement than civilians; their habits of discipline and obedience, and the pride, and emulation which may so easily be excited amongst them, render them much more docile and improveable as a body, than any other class of men, provided their instruction is carried on under the eye of superiors zealous in the cause.

The statements, which have just been made, may plead as to the absolute necessity of endeavouring to improve the non-commissioned officers and soldiers of the Royal Engineer department, in order that they may be able to render more effectual assistance to their officers in the field. And if the remarks upon the causes of their comparative ignorance, and consequent inefficiency, are allowed to be just; it must also be admitted as a natural inference, that there is no possible mode of collecting, forming, and keeping up a body of well-educated and efficient military artificers, except by instructing them, according to some properly digested system, after they enter his Majesty's service. The present Course of Instruction having been composed for this express purpose; a few words may be said upon the form, which has been given to it.

It is a very natural supposition, which may probably occur to any well-educated person, when he first takes up this book; that it might have been more advantageous in a system of Practical Geometry, professedly intended for the use of uneducated men, to have greatly curtailed the definitions and problems; and to have introduced vulgar terms and phrases in place of the regular terms of the art.

Reflection, founded upon experience, convinced the author, who was at first inclined to embrace this opinion; that any attempt, thus to simplify, would only impede the progress of the learners, and would in a great measure defeat the advantages which are to be expected from a more regular system.

In the remarks on the Supplementary Problems, which the Teacher is supposed to deliver at the board, strong reasons are given for adhering to the proper definitions and other geometrical terms. There is another reason still stronger.

Unless these terms are strictly adhered to, the officers and men of the Royal Engineer department will never be able to act in proper concert, or even to understand each other, without a great waste of valuable time spent in unnecessary and troublesome explanations.

For instance, supposing that any line is marked on the ground, and that you order a man to lay out a second line parallel or perpendicular to it: if he understands the meaning of these terms, he will do it at once without giving you further trouble; but if he is an uninstructed man, it will be impossible, or next to impossible for you, to explain your wishes, by means of words alone. You will find no other method of making him comprehend you, but by actually marking out the second line yourself. Whilst you are thus employed, you waste your own time, and lose the services of the person whom you are vainly endeavouring to render useful; and, what is worse, some hundreds of men whose exertions might be of the utmost importance, are probably obliged to stand idle in consequence. Whoever chooses to try the experiment, may soon satisfy himself, that it is no easy task to deliver himself in such language, as will make an uneducated person comprehend what he means by the words parallel and perpendicular.

It will by no means be sufficient, if an officer of engineers makes himself master of the vulgar expressions, which may be supposed more adapted to the previous education and capacity of the men. These terms are generally much too vague to give any precise notion of what is intended. The better class of civil artificers, as was before remarked, understand geometry; and you will therefore find no difficulty in communicating your ideas to them: but nothing can be more confused than the language of uneducated workmen; even of such as may have attained considerable practical dexterity. For instance, amongst men of this description, the word SQUARE, when applied to a line, generally signifies a perpendicular: sometimes, however, they use it in speaking of a parallel: in addition to which, the same word has been employed not only to denote a mason's or carpenter's instrument so called, but also a geometrical square, a rectangle; and even a right-angled triangle. What is to be done with such a jargon?

In the above Course of Practical Geometry, some problems and definitions have been included for the sake of making the work complete, which may not appear absolutely necessary, considering the object in view. The officer directing, or Teacher carrying the Course into effect, may therefore curtail, according to their judgement, whatever they consider superfluous. But, excepting the scales of sines, tangents, and secants, near the end, there is perhaps nothing which it would not be of advantage to the learners to perform, if time and circumstances permitted; because after going through a Course of this extent, their understanding would be so much improved, and their confidence in their own powers so much increased; that no plan of any kind of military work, in which they are likely to be employed, afterwards, will be beyond their comprehension, or will even appear difficult to them.

If the problems towards the end of the Course, which at first sight may appear rather complicated; were too hard a task for men of common capacity, it might certainly be advisable to dispense with them. But this is not the case. Experience has proved that the learners after mastering the first nine or ten simple problems, are able to go through the remainder of the Course with equal if not with greater facility; so that their progress in improvement is generally more rapid, in proportion as the Course draws towards a close.

In particular cases only, when the time allotted for instruction is too short to admit of putting the learners through the whole course of Practical Geometry, then the Teacher, after grounding them well in the most necessary definitions and problems, may pass over the remainder of the Course, and make them commence the practical part of Plan-Drawing at once.*

Should the men even go no further in this latter branch, than to draw by scale a few sections of some simple works, such as they may have an opportunity of seeing, at the station where they are quartered, it will be of the greatest advantage. A man who understands this much, may always be made useful in the Royal Engineer department, and there is an opening for his further improvement, whenever a new opportunity offers.

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* It will, however, always be advisable to go on regularly, at least as far as the end of Problem 33 (in page 71), before Plan Drawing is attempted.

Amongst a number of men indiscriminately collected together, there will always be some individuals of weak understanding, incapable of making any great progress in learning. Such men ought to be discharged from the classes of Practical Geometry, and employed in duties requiring personal exertion only.

At the first commencement of instruction, every man, however, excepting those who cannot write, ought to have a fair trial; and no individual ought to be objected to, merely because he writes badly. The ignorance of that class of men, who enlist as private soldiers, generally proceeds from the poverty or avarice of their parents, who have been unable or unwilling to educate them; and is therefore no proof of want of ability. On the contrary, it has been observed, repeatedly, that some men who could scarcely write legibly, have, in consequence of the superiority of their natural talents, made much greater progress in Practical Geometry and Plan Drawing, than others who were previously much better educated, and who could write very good hands.

The comparative abilities of men learning Practical Geometry will be discovered in the course of the first eight or ten lessons; after which, the further attendance of those, whe prove incapable of learning, should be dispensed with; but the proportion of such men will always be very small.

Those men also, who abandon themselves to drunkenness and dissipation, or who betray bad principles, should never be allowed to attend the classes of Practical Geometry, even if they are known to possess respectable or superior abilities. This Course of Instruction has by no means been drawn up with a view to the individual advantage or improvement of the persons instructed; but solely with a view to the benefit of the service. It is intended to qualify the non-commissioned officers and soldiers of the Royal Engineer department, for performing services, in the field, of a most important nature, which require a combination of knowledge, zeal, and fidelity. But men of weak capacity will never be able to do justice to such duties; and men of vicious habits or bad principles are not to be trusted in the hour of danger and difficulty, however great their abilities may be. It is highly desirable, that some regulation were made, by means of which men of the above description could be excluded or expelled from the Royal Engineer department; a branch of the service, which ought to be select in regard to the qualifications not only of the officers, but of the soldiers who belong to it.

END OF THE

COURSE OF PRACTICAL GEOMETRY.

THE
PRINCIPLES
OF
PLAN DRAWING.

BEFORE the Learners begin to study this part of the Course, it may be advisable to exercise them in drawing a few plans and sections of a simple nature; for some previous practice of this kind will enable them to comprehend the principles of the art of Plan Drawing with greater facility.

The Teacher is supposed to deliver the following instructions, from the board, in the usual manner.

I shall now proceed to explain to you the Principles of Plan Drawing.

Drawings are figures delineated or laid down on some plane surface, such as paper, in order to explain the form and dimensions, or to represent the outward appearance of any given object.

There are different kinds of drawings, executed according to different principles and for different purposes.

The art of Plan Drawing includes what are called geometrical drawings only.

Geometrical drawings are those which are capable of explaining the true proportions and dimensions of any object.

The objects represented in drawings are generally solid bodies, or irregular curved superficies; but it must be evident, on a very little reflection, that any one single figure or view cannot give a just notion of the dimensions and appearance either of a solid, or of a curved superficies.

For instance, if you place yourself in front of a house, or opposite to one end of it, or if you stand behind it, or look down upon the roof of it from some great height, such as the top of a lofty steeple, you will have a different view of it in all these cases, so that unless you take several drawings of it from different points, it will be impossible for you to give any just notion of the general appearance of the building.

There are only three kinds of geometrical drawings necessary for explaining the nature of any object: namely plans, sections, and elevations.

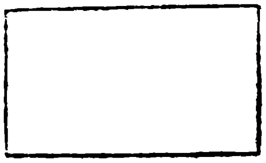
Write the word PLAN.

A plan of any object nearly resembles the appearance, which the object would have if it could be viewed from a point above it.

In order to illustrate this more clearly, we shall proceed to draw the plan of a small building.

In commencing a building, the first thing necessary is to have a ground plan, or plan of the foundation. Let us suppose that the building, which we are going to represent, is a cottage with a door and one window only.

You will first draw a rectangle to represent the exterior dimensions of the cottage, that is to say its dimensions from out to out.

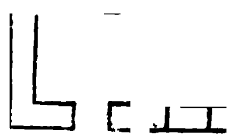


The thickness of the walls must next be represented. This is done by drawing four lines parallel to the sides of the rectangle.

You will complete your walls accordingly.

You now see that your figure has nearly the same appearance, which a small building would have, if it could be viewed from any point immediately over it, after the foundation was begun.

Doors and windows are generally marked in a ground plan. In order to distinguish them from each other, those lines of the foundation or walls which interfere with a door are rubbed out. I shall mark my door and window accordingly.



You will follow my example.

You have now drawn the complete plan of the cottage. It shows the size of the room; the thickness of the walls; and the width and position of the door and window.

By means of a plan done according to scale, it would be easy for you to lay out correctly the foundation and door of a small cottage, such as is represented in your present figures; but after building a few courses, you would be obliged to stop for want of further directions, because the plan can neither explain the height of the door or window, nor the height of any other part of the building.

This agrees with what I before stated, namely that more than one kind of drawing of any object is always necessary: but before we proceed to the other kinds of geometrical drawings, above mentioned, I must add some further observations and explanations respecting the nature of plans.

The plan of any object is always supposed to be laid out upon a horizontal plane or dead level.

The necessity of following this rule may be understood from the following consideration.

Supposing that you wanted to build a house on uneven ground, such as the side of a hill, every one knows that in laying out the foundation, you could not trust to any oblique measurements made along the slope.

For instance, if you were to measure 30 feet obliquely along the side of the hill for the breadth of your proposed building; it would of course be necessary to level the ground, before you could lay the first floor. After this was done, you might find the space which you had laid out for the breadth of your building, reduced from 30 feet to 28 feet, or 25 feet, or even less, in proportion to the steepness of the original slope of the hill.

The plan of any uneven field, in which the dimension were marked according to oblique measurements made upon the sloping or irregular surface of the ground, would therefore be of no use.

To draw the plan of any work or object, having sloping or oblique planes and lines, the dimensions of which must consequently be reduced in a certain proportion, for the reason above stated, is more difficult in practice than the plan of a building; but the principle upon which it ought to be done is sufficiently simple.

The rule for laying down the various points of any oblique object truly upon a horizontal plane, which every plan is supposed to be, is as follows.

The horizontal plane, to which the various points of any object must be transferred, in order to find their proper place in the plan, may either be supposed to cut the given object, or to lie immediately below or above it.

When you come to understand the principles of Plan Drawing, which I am now about to explain to you, you will be at no loss to apply these principles properly, in either of the above suppositions.

But for learners it will be clearer to begin by supposing the horizontal plane to pass through the base or lowest point of the given object.

This imaginary horizontal plane, is to be your guide for drawing the plan of the given object; and in geometrical drawings, any plane supposed to be used for this purpose is called a plane of projection.

Write the words PLANE OF PROJECTION.

In respect to such points of any object as stand upon the plane of projection, or which coincide or agree with it, it must be evident that these points can give you no further trouble, the spot where they actually stand being their true place in the plan.

From every point which does not coincide with the plane of projection, a perpendicular is supposed to be drawn to the said plane; this perpendicular will mark upon the plane of projection the true position of the point from whence it is drawn.

If the plane of projection is supposed to lie below the given object, then the various points of the object will be above the plane. Consequently all the perpendiculars requisite for finding the position of these points on the plane of projection, must be dropped from the said points.

But if the plane of projection is supposed to be above the given object, then the various points of the object will be below the plane. Consequently all the perpendiculars requisite for finding the position of these points on the plane of projection, must be raised from the said points.

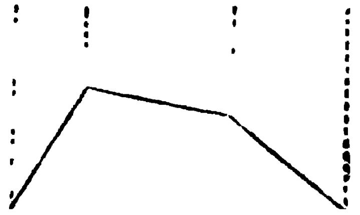
The plane of projection, in plans, is always supposed to be a horizontal plane, as has been before observed; but every perpendicular to a horizontal plane, no matter whether dropped or raised, must be a vertical or plum-line.

Consequently, if you suppose two plummets to be suspended exactly over any two points of an object, the plan of which is required to be drawn; the distance between the plum-lines, measured perpendicularly, not obliquely, will be the true distance at which the above points ought to be laid down on the plan.

To explain this, I shall draw three oblique lines on the board, connected together, all of the same length, but sloping / unequally. These will represent the form of some sloping or oblique object, of which a plan is to be drawn.

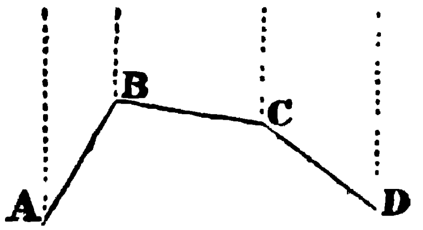
You will copy this, and the following operations upon your slates, without further directions, until the figure which I have now begun is completed.

From the extremities of each of the three lines, I shall now draw dotted lines, parallel to each other, directed towards the top of the board.

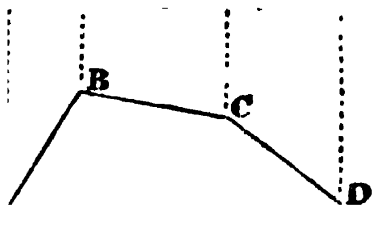


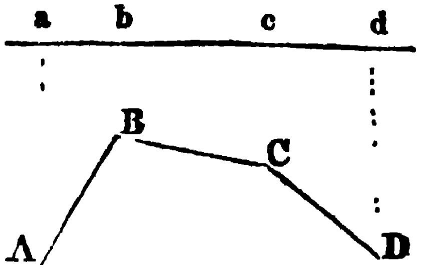
These dotted lines may represent plum-lines held over the various points of the oblique object.

I shall mark the various points of my oblique object by the capital letters A, B, C, and D, from left to right.



As the distances between the four plum-lines represented in my present figure, must be measured perpendicularly, not obliquely; I shall draw a line above the given object, perpendicular to the dotted lines, to represent the said distances.





At the points, where the above perpendicular is intersected by the dotted lines, I shall mark the small letters a, b, c, and d, from left to right.

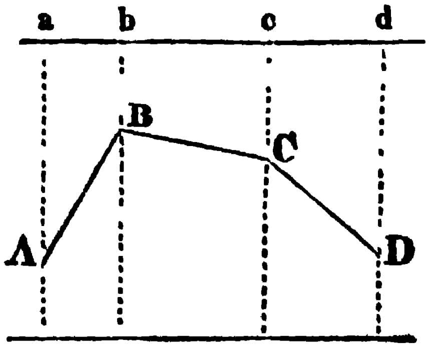
The distance between the points a and b, at the top of my figure, now represents the exact distance measured perpendicularly, which there would be between two plum-lines suspended over those points of the given object which are marked by the capital letters A and B.

Consequently the perpendicular line a b at the top of the figure represents the exact length, which ought to be given to the oblique line A B, in drawing a plan of the given object.

The perpendicular line b c at the top of the figure, in like manner and for the same reason, represents the exact distance which ought to be given to the oblique line B C, in drawing a plan of the given object.

And the perpendicular line c d at the top of the figure, in like manner and for the same reason, represents the exact distance which ought to be given to the oblique line C D, in drawing a plan of the given object.

I shall now produce the dotted lines downwards, below the given object: and I shall draw a new perpendicular intersecting the dotted lines produced.



I shall also mark the various points of intersection upon this new perpendicular, by the small letters a, b, c, and d.

Then as parallel lines are always at the same distance from each other although produced ever so far, the distance between the points a and b, at the bottom of the figure, will be equal to the distance between the corresponding points a and b, at the top of the figure.

The distance between the points b and c, at the bottom of the figure, will be equal to the distance between the corresponding points b and c, at the top of the figure.

And the distance between the points c and d, at the bottom of the figure, will be equal to the distance between the corresponding points c and d, at the top of the figure.

Consequently the perpendicular distances a b, b c, and c d, at the bottom of the figure, will be equal to the perpendicular distances a b, b c, and c d, at the top of the figure: and the lines a b, b c, and c d, at the bottom of the figure, may therefore serve equally well to represent the respective lengths which ought to be given to the oblique lines A B, B C, and C D, in drawing the plan of the given object.

As the dotted lines represent plum-lines or vertical lines, the lines at the top and bottom of the present figure, being perpendicular to them, must necessarily be level or horizontal lines; and, in a drawing, they may represent part of a horizontal plane.

Consequently either of these two lines may represent a plane of projection supposed to be used in drawing the plan of the oblique object: for instance, the upper line may represent a plane of projection supposed to pass above the given object: and the other may represent a plane of projection supposed to be below it.

In both cases, the dotted lines may represent perpendiculars, which, as I said before, are always supposed to be drawn from the various points of an object, to the plane of projection, in order to find the true place where the said points ought to be laid down upon a plan.

This illustrates what I before observed, that in drawing the plan of any thing, it makes no difference whether the plane of projection is supposed to pass above or below the given object.

A perpendicular drawn from any point in a given object to a plane of projection, in order to determine the true place of the said point for a plan or geometrical drawing, is called a ray.

Write the word RAY.

When the word ray is used in place of perpendicular, then a ray is not said to be drawn from any given point, but to be thrown out or projected from it.

The spot where a ray, thrown out from any point of a given object, strikes the plane of projection, is called the projection of that point.*

And as all the figures, which represent oblique objects in Plan-drawing, are found by means of rays projected, from the various points of the said object: such figures are often called geometrical projections.

And the rules or principles of Plan-drawing, which I have now partly explained to you, are sometimes called the Principles of Projection.

Having explained these terms, I shall make some further remarks on our present figure.

If we suppose a dyke, or embankment of earth, to be cut down perpendicularly, and that the figure formed by the oblique lines A B, B C, and C D, represented the vertical section of it after it was thus cut; then the dotted lines a a, b b, c c, and d d, show the manner in which it would be necessary to represent a part of the said dyke, in drawing the plan of it.

In the section you see that the lines A B, B C, and C D, are all equal to each other if measured obliquely according to the slopes, whilst the lines marked with the small letters a b, b c, and c d, which represent the breadth of the above slopes upon the plan, are all shorter than the former and all unequal, being shortened more or less exactly in proportion to the steepness of the slopes.

If the middle line of the dyke B C instead of being oblique had been perpendicular to the dotted lines: then it would have been equal and parallel to the corresponding line also marked b c, at the top or bottom of the figure which represents its length in the plan.

Consequently in geometrical drawings, those lines of any given object, which are parallel to the plane of projection, may be laid down in their actual dimensions according to measurement, without any alteration or diminution.

Rub out your last drawn figure, leaving only the plan of the cottage.

Before we proceed further, I shall give you another illustration of the above rule by means of a square pyramid.

Here the Teacher will produce a square pyramid.

The plan of any thing nearly resembles the appearance which the object would have if it could be viewed from a point above it, as was before stated; but if you look down upon a square pyramid in this manner, you will see the extremities of its base; the vertex of it; and the four ridges or oblique lines, which are formed by the meeting of its sides.

All these particulars must therefore be represented in the plan of a square pyramid.

According to the rule, which I before said was the most convenient, I shall suppose the horizontal plane or plane of projection, which is to guide me in drawing my plan, to pass through the base of the pyramid.

For instance, if I place my pyramid upon a table thus *(Here the Teacher will exemplify)*, the level surface of the table will represent the plane of projection.

The base of my pyramid stands upon the plane of projection, and coincides with it. Consequently the base of the pyramid may be laid down in a plan, in its actual proportions, without any alteration or diminution.

The base of my pyramid is a square. I shall accordingly represent it on the board, by drawing a square exactly equal to it.

Here the Teacher will exemplify.

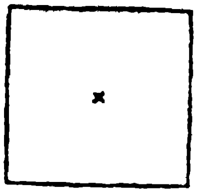
The four ridges or oblique lines remain to be drawn: but one extremity of each of these agrees with the angles of the base, all of which are already marked on the plan.

The other extremities of these ridges meet in the same point, namely, at the vertex of the pyramid: as soon as the proper position of the vertex is found, there will therefore be no difficulty in completing the plan.

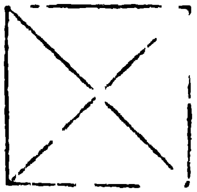
For this purpose a perpendicular, vertical, or plum-line, is supposed to be dropped from the vertex to the plane of projection.

In the present instance, the base of the pyramid and the plane of projection agree with each other: and as the pyramid is perfectly regular, it must be evident that a perpendicular or vertical line dropped from the vertex would exactly fall upon the middle or central point of the base.

I shall therefore find the middle point of my square and mark it, in order that it may represent the vertex of the pyramid.



From this point I shall next draw a right line to every angle of my square.



The four last drawn lines will represent the ridges; and the figure in its present state is a true plan of the square pyramid, on the same scale as the pyramid itself.

I chose a square pyramid partly because it is a very simple figure, and partly because it explains the principle upon which the plan of the roof of a building ought to be drawn; a plan of this kind being often necessary in addition to a ground plan.

From the plan of the pyramid, the same inference may be drawn, as from the former plan of the cottage.

You see that the plan of the pyramid, now represented on the board, shows nothing more than the dimensions of the base. It also shows the particular point of the base over which the vertex would stand, but it can neither explain the height of the pyramid, nor the obliquity or slopes of its sides.

Therefore a plan alone cannot explain the nature either of a building, of a pyramid, or of any other object, without the assistance of some other kind of geometrical drawing, such as a section or an elevation.

Here the Teacher will rub out his plan of a pyramid.

I shall next explain to you the nature of sections.

A section is the plane figure which would be formed by cutting any solid body right in two, as was defined, in the foregoing Course of Practical Geometry, in treating of conic sections.

Write the word SECTION.

A solid body may be supposed to be cut in a great number of various directions, horizontally, vertically, and obliquely; and consequently the number of sections which may be taken of any object are almost infinite or beyond calculation.

To prevent the confusion, which would arise in Plan-drawing, from sections taken at random; the geometrical drawing, called a section, is always supposed to be taken vertically.

That is to say, the object is supposed to be cut right down, perpendicularly, from top to bottom, by a vertical plane; or in other words, it is supposed to be cut every where according to the plum-line.

The reason of following this rule will be obvious on a little reflection.

A section is principally intended to explain the heights of objects, and thereby to make up for the deficiencies of that kind of geometrical drawing called a plan, which has just been explained.

But supposing I wanted to measure the height of one of the sides of this room; you must all be sensible, that if I took my measurement diagonally or obliquely, it would be quite wrong; and that there is no way in which the height of a room can be measured truly, unless it is done vertically or according to the plum-line.

If you suppose, therefore, a section of this building to be taken, and that the said section passed through the room in which we now are; it must be evident that if the section was taken in a sloping direction, it would also cut the side of the room obliquely.

Such a section would therefore give an erroneous representation of the height of the room.

Sections taken across any building or other object, will of course serve to show the breadth as well as the height, of the various parts of it. In order that this may be done truly, another rule has been laid down no less essential than the former.

The rule now alluded to is as follows.

In taking the section of any regular object, such as a rectangular building, the given object is always supposed to be cut right across, not only vertically, but also in a direction perpendicular to two opposite sides of it.

The same reason holds good in this case, which applied to the former rule.

Supposing that I wished to measure the breadth of this room, you must be aware that if I took my measurement obliquely or diagonally from one angle of the floor, towards a contrary one; the result would be quite wrong; and that there is no possible way of measuring the breadth of the room accurately, except in a direction perpendicular to the two opposite sides of it.

But if you suppose, as before, a section of this building to be taken, and that the said section passed through the room in which we now are, it must be evident that unless the section was taken in a direction perpendicular to the opposite sides of the building, it would cut the floor and ceiling of this room obliquely. Therefore such a section would give an erroneous representation of the breadth of the room.

From these considerations, it must now be evident, that any section of an object, taken in a sloping or oblique direction, would not be of the smallest use, because it would either misrepresent the height, or the breadth, of the given object, or both.

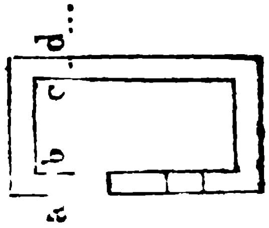
This being premised, we shall now proceed to draw a section of the small cottage, of which we have already drawn the plan.

Let us suppose, that the proposed section is required to pass through the door of the building.

Draw a dotted line perpendicularly across the plan of your cottage, passing through the door.

This dotted line will represent the direction in which the proposed section is to be taken.

Mark those points in the plan where the dotted line cuts the front and back walls of the cottage by the letters a, b, c, and d.

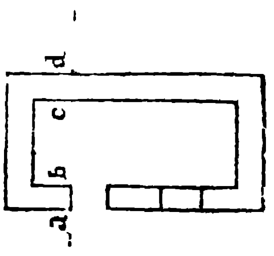


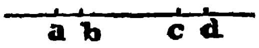
The distances between the various points a, b, c, and d, which you have just marked in the plan, show the breadth of the cottage, and the thickness of the walls.

As the same dimensions will require to be represented in the section, it will save trouble to transfer the whole of them from the plan to the section, at once.

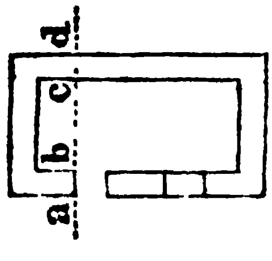
You will therefore draw a separate line to represent the level of the floor of your building, which is also to be the ground line or base of your section. And you will divide this line exactly in the same manner as the dotted line in the plan.

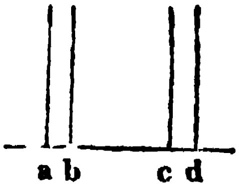
Under the respective points of division, on this new line, you will mark the same letters a, b, c, and d: when this is done, the corresponding or equal parts of both lines will be known by inspection.





From the points marked a, b, c, and d, on the ground line of the section, which represent the position and thickness of the walls of the cottage, raise perpendiculars to show the height of the walls.

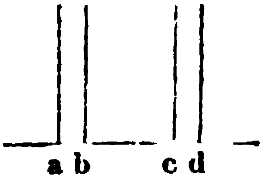




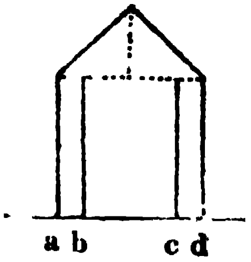
Join the top of these perpendiculars by a dotted line, which will be a horizontal line; and which shows the level from whence the roof is supposed to spring.

The plan of the cottage is still supposed to remain on the board, and slates, but it is left out in some of the following figures. It will again be occasionally introduced, whenever it shall become necessary to point out the connection between the plan, and the section or elevation.

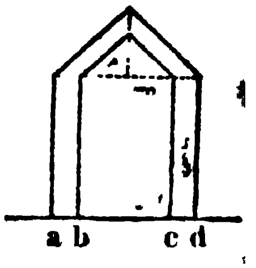
We shall suppose the roof to be a regular pitch roof. You will therefore bisect the last drawn line, in order to find the middle of the building, and from the point of bisection, you will raise a perpendicular to show the height of the roof.



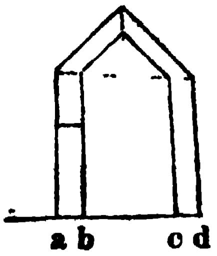
You will next draw oblique lines to show the sides of the roof.



Draw right lines parallel to the above oblique lines to show the thickness of the roof.



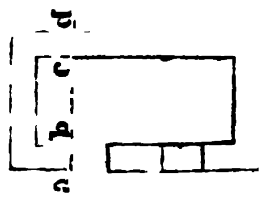
As the section is supposed to pass through the door of the cottage, a line must be drawn to represent the top of the door, and to show the height of it.

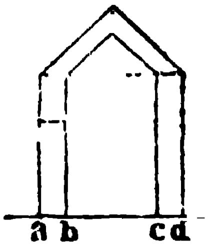


The section, which has just been drawn, is only intended to give you a general notion of this kind of geometrical drawing. Many particulars are therefore omitted which it would be proper to introduce in a finished section of a building.

For instance, the depth and thickness of the foundation, the rabbet or recess of the door, the thickness of the rafters and other parts of the roof, and the projection of the roof, if formed with eaves: these and other details might easily have been represented, by adding a few more lines.

The rough section of the cottage is now complete, and you may observe that those dimensions which are marked with the same letters agree in both.





Rub out the letters which were written to mark the various points in the plan and section.

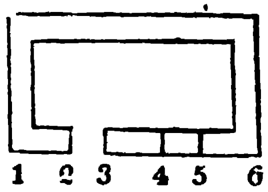
The plan and section as they stand at present, explain sufficiently the general dimensions of the shed, and the proportions of the roof and door, but they do not show the height of the windows, nor the general appearance of the building.

The latter particulars cannot be represented without the assistance of the third kind of geometrical drawing, before mentioned, called an elevation.

Write the word ELEVATION.

An elevation is the view of any upright side of a building or other object, nearly such as it would appear to a person standing exactly in front of it.

In order more clearly to understand this definition, we shall proceed to draw an elevation of the front of our shed.

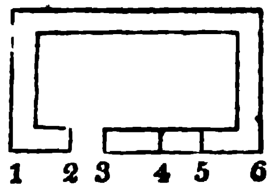


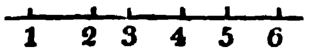
As the principal dimensions of the front of the cottage appear in the plan, I shall mark the various points by the numeral figures 1, 2, 3, 4, 5, and 6.

The points thus marked show the length of the front of the cottage, and the breadth and position of the door and window.

As the whole of these dimensions must appear in the elevation of the cottage, the easiest method will be, to transfer them from the plan to the elevation at once.

You will therefore draw a separate line, to represent the ground line or level upon which the front of the cottage stands; and upon this line you will set off a distance equal to the length of the cottage, and divide it in the same manner as the front of the cottage is divided in the plan.

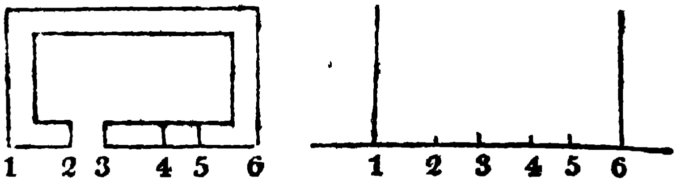




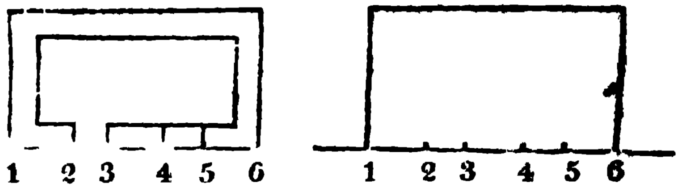
You will also mark the various points of division upon this new line, with the numeral figures 1, 2, 3, 4, 5, and 6. When this is done, the corresponding or equal parts in the plan and in the ground line of the elevation may be known by inspection.

From the points 1 and 6 of the ground line which represent the extremities of the front of the cottage, perpendiculars must next be drawn to show the height of the walls.

You will draw two perpendiculars accordingly; and as the height of the walls is already represented in your section, you will measure the proper length of these perpendiculars from thence.



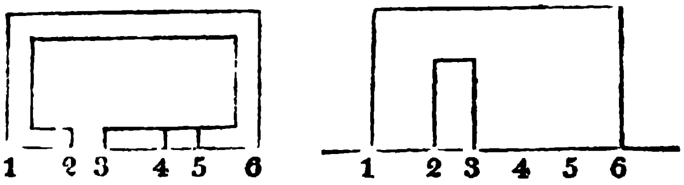
Join the top of these perpendiculars by a right line, which will represent the bottom of the roof of the cottage.



From the points 2 and 3 of the ground line of your elevation, which represent the breadth of the door, you will raise perpendiculars, to show the height of the door.

You will find the proper length of these perpendiculars by measuring the height of the door in the section; and you will transfer it to the elevation accordingly.

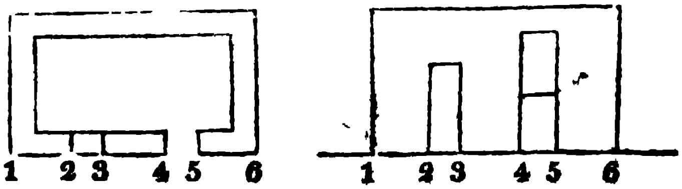
Complete the form of the door by joining the top of the above perpendiculars.



From the points 4 and 5, in the elevation, which mark the position of the window, raise perpendiculars to find the sides of the window.

You will next complete the window by drawing the top and bottom of it, at any height you think proper, these particulars not being represented in the section.

Dot that part of each of the last drawn perpendiculars, which falls below the bottom of your windows.



The form of the roof only is now wanting. The length of the roof must of course be equal to the length of the building, and the height of it may be found by referring to the section.

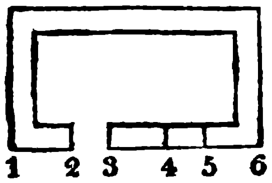
It is a rule in geometrical elevations never to represent the height of any sloping object by oblique measurements taken along the slope; but by dropping a perpendicular from the highest point or vertex of the slope, to the level of the lowest point or base of it.

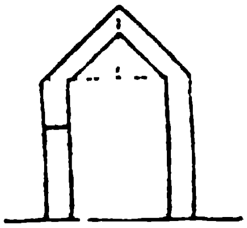
In short, the height of any sloping object in a geometrical elevation is measured by that perpendicular line, which would be called the altitude of any similar figure or solid, in Practical Geometry.

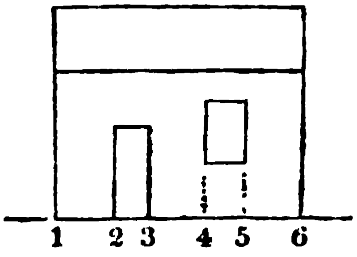
In transferring the height of the roof from your section to your elevation, you must therefore make it equal to the dotted perpendicular, which appears in the section.

Draw your roofs accordingly.

When this is done, the figures upon the board, which relate to the cottage, will be as follows.







The plan, section, and elevation of a small cottage are now complete, and from these three geometrical drawings put together, every dimension necessary for explaining the proportions of the building may be known.

The length of the building is shown in the plan and elevation; and is the same in both.

The breadth of the building and the thickness of the walls are shown in the plan and section; and are the same in both.

The breadth of the door and that of the window are shown in the plan and elevation, and are the same in both.

The height of the door is shown in the section and elevation, and is the same in both.

The height of the window is shown in the elevation only, but if the section had been taken through the window, instead of the door, then the height of the window would have been shown in the section also.

The height of the walls, and the perpendicular height of the roof are shown in the section and elevation, and are equal in both.

But the particular form of the roof is clearly explained in the section only.

I shall now make some further remarks relative to geometrical elevations.

In drawing the geometrical elevation of any object, a vertical plane is always supposed to be used as a plane of projection.

Some points of the given object may coincide or agree with the plane of projection.

Those points of the given object, which fall without the plane of projecction, must be transferred to it, by throwing out perpendiculars or rays, from the said points to the plane of projection.

All the rays in a geometrical elevation, being perpendicular to a vertical plane, must necessarily be horizontal or level lines.

The walls or sides of a building are vertical planes, being built according to the plum-line ; and therefore, in taking geometrical elevations, the front of a building, or any other side of it which is to be represented, may be supposed to agree with the plane of projection.

Consequently the length and height of the side of the building, and the height and breadth of the doors and windows, &c. may be laid down in a geometrical elevation, according to their actual dimensions from measurement, without any alteration or diminution.

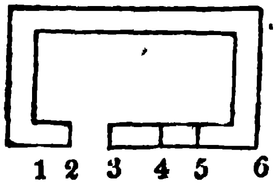
The roof from its sloping figure is the only part of the exterior side of a building, which cannot agree with the plane of projection; and therefore, in drawing the elevation of the cottage, I made you diminish the oblique length of the slope of the roof, in order to find the true height of it, according to the same principle by which oblique lines are diminished in a plan, in order to find the base of any slope.

It is not necessary in a geometrical elevation, that the plane of projection should be supposed exactly to agree with the upright side of any building or object, which is to be represented.

But when they do not agree, it is necessary that the plane of projection should be parallel to the upright side of the building or object, of which an elevation is to be drawn.

In that case the perpendiculars or rays, thrown out from the object to the plane of projection, will form a figure exactly in the same proportion,' as if the side of the object and the plane of projection had actually coincided with each other.

This may be understood by referring once more to the plan.

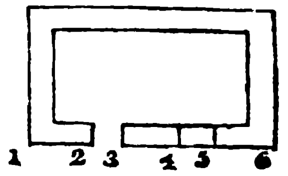


In the way, in which we actually drew our elevation, the plane of projection was supposed to coincide or agree with the front of the building, and the various points 1, 2, 3, 4, 5, and 6, which represented the length of the said front, and the breadth and position of the door and window, were therefore transferred from the plan to the elevation, at once, according to measurement.

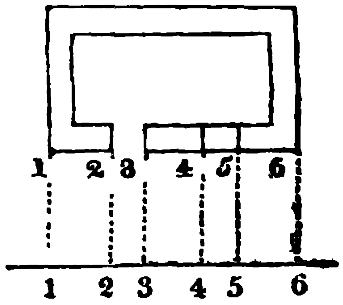
Now I shall suppose the plane of projection to be at some distance from the front of the building, but parallel to it, according to the rule above stated.

The sides of the building which are vertical planes being represented in a plan by right lines, any other vertical plane, whether parallel to the front of the building or not, must also be represented in the plan, in the same manner.

You will therefore draw a right line in the plan, parallel to the front of the building, to represent a plane of projection, at some distance from the building, but parallel to the said front.



From the various points 1, 2, 3, 4, 5, and 6, of the front of the building, you will drop dotted perpendiculars to the line which represents the new plane of projection; and you will also mark the points where each of these perpendiculars meets the plane, by corresponding numbers 1, 2, 3, 4, 5, and 6.



These dotted perpendiculars represent rays thrown out from the given object; and the points, where they meet the plane of projection, are true points for laying down the various dimensions, in the elevation.

But in consequence of the plane of projection and front of the building being parallel, the above dotted lines or rays which are parallel to each other, will be perpendicular not only to the line which represents the plane of projection, but also to the front of the building.

The distance between parallel lines is always the same if measured perpendicularly, therefore the distance between the points 1 and 2 on the front of the building, will be equal to the distance between the corresponding points 1 and 2, on the plane of projection:

The distance between the points 2 and 3 on the front of the building will be equal to the distance between the corresponding points 2 and 3 on the plane of projection:

And in like manner, the distance between any two points whatever, on the front of the building, will be equal to the distance between two corresponding points, on the plane of projection.

This proves, what I before stated, that the perpendiculars or rays, thrown out from the upright side of an object to any parallel plane of projection, will form a figure exactly in the same proportion, as if the side of the object and the plane of projection had actually coincided or agreed with each other.

Consequently, if you suppose a plane of projection to be chosen, parallel to the upright side of a building, or other object, of which an elevation is required; then the dimensions of the various parts of the upright side of the given object may be laid down in the drawing, in their true proportions, according to measurement, without any diminution or alteration.

From the figures which you have drawn, and the instructions which you have already received upon the subject of Plan-drawing, you must now be aware, that plans and elevations are drawn exactly according to the same principle; only, that in a plan, the plane of projection is horizontal; whereas, in an elevation, the plane of projection is always vertical.

All horizontal planes, which may be used as planes of projection for drawing the plan of a building, &c. must be parallel to each other; but vertical planes may be perpendicular or oblique to each other, as well as parallel.

For instance, the several floors of any building being all level, and of the same height one above another at every point, are horizontal planes parallel to each other. But of the walls of a building, which are all vertical planes, some two of them may be perpendicular to each other, such as the side and end walls; whilst others may be oblique to each other, as you may often have observed in irregular buildings.

If in drawing the elevation of any rectangular building, the plane of projection were chosen oblique to one of the sides, instead of parallel to it; then the length of that side of the building, and the breadth of the doors and windows, &c. would be diminished in the drawing, in such a manner, as to give a false notion of the object.

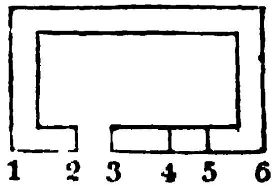
In an oblique elevation of this kind, rays thrown out from one of the ends of the building would also strike the plane of projection, and the various dimensions of this end of the building would be diminished or misrepresented in the drawing, for the same reason.

Every dimension, excepting only the perpendicular heights, being misrepresented in an oblique elevation of the side of a building, or of any other upright object; such elevations can be of little use, and are therefore seldom or never taken.

But although oblique elevations of the front of a building, &c. are seldom or never taken, it often happens that the front of a fine building is ornamented with columns, mouldings, and other architectural decorations, various parts of which are oblique to the general plane of the front of the building, beyond which they project.

The proper mode of representing such ornaments in geometrical elevations, cannot therefore be well understood, unless the principle, according to which oblique elevations of any upright object may be drawn, is clearly explained.

This being premised, I shall proceed to give you an example of the method of drawing an oblique elevation of the cottage, of which you have already drawn the plan, and section, and a direct or proper geometrical elevation.

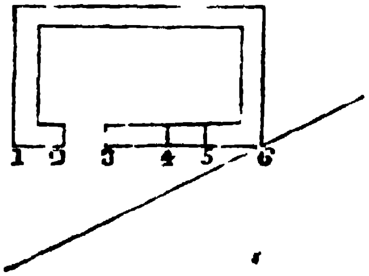


Rub out that line which was added to your plan to represent a parallel plane of projection: rub out also the dotted perpendiculars which represent the rays; and the plan will be restored to its former state.

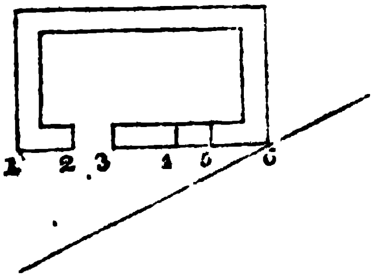
A right line must next be drawn, in order to represent the new plane of projection which is necessary for taking the oblique elevation.

This new plane of projection may either be supposed to coincide with the front of the building in some point, or not.

We shall suppose it to coincide or agree with that extremity of the front of the building, which is marked by the numeral figure 6.

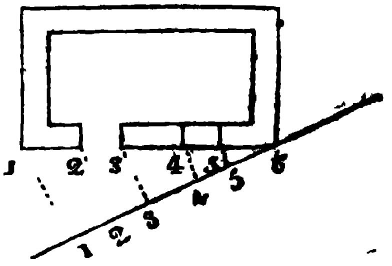


You will therefore draw a right line through the point 6, forming an acute angle with the front of the building, in order to represent the new plane of projection.

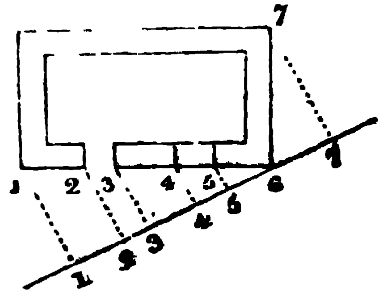


From the various points of the front of the building drop perpendiculars to the last drawn line, and dot them.

These perpendiculars will represent rays, thrown out from the points 1, 2, 3, &c. of the given object to the plane of projection, in order to find the true place of these points in the oblique elevation.



You will therefore mark, in like manner, the various corresponding points on that line which represents the plane of projection, by the same numeral figures 1, 2, 3, &c.



That end of the cottage, which is nearest to the plane of projection, must also be represented. One extremity of it already coincides with the said plane. From the other extremity of it, you will draw a dotted perpendicular or ray to the plane of projection, and you will mark the corresponding points at the ends of this ray by the numeral figure 7.

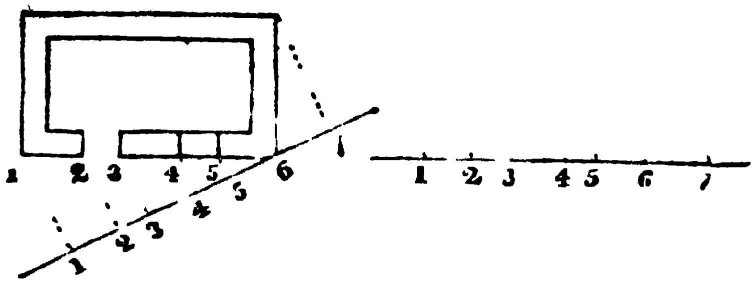
The distance between the points 1 and 6, in the plan, shows the length of the front of the cottage.

The distance between the corresponding points 1 and 6, on the plane of projection, will therefore also represent the length which ought to be given to the front of the cottage, in the oblique elevation.

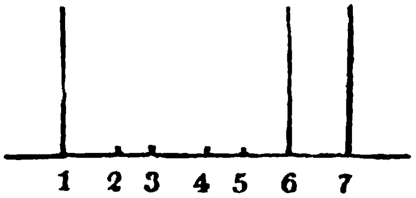
The distance between the points 6 and 7, in the plan, represents one end of the cottage.

The distance between the corresponding points 6 and 7, on the plane of projection, will therefore also represent the length which ought to be given to that end of the cottage, in the oblique elevation.

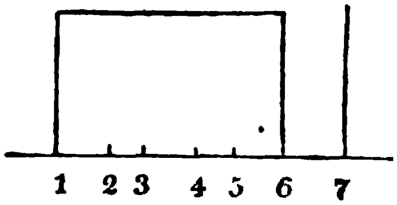
And, in like manner, as the breadth of the door and window are represented by the distance between certain points, in the plan; so must the same dimensions, in the oblique elevation, be represented by the distance between corresponding points, on the plane of projection.



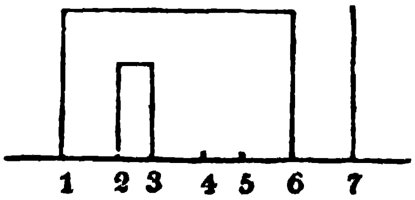
You will therefore draw a line for the ground line of your oblique elevation; which you will divide in the same manner, and mark with the same numeral figures, as that line which represents the plane of projection.



From the points 1, 6, and 7, on the ground line of your elevation, you will raise perpendiculars equal to the height of the walls of your cottage. The length of these perpendiculars will be determined by referring to the section, or to the former elevation.



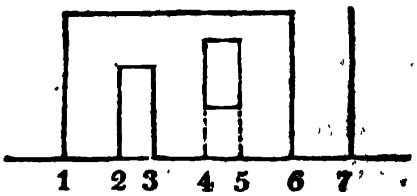
You will join the top of two of these perpendiculars by a right line, in order to show the bottom of the roof of the front of the cottage.



By means of the points 2 and 3, which determine the position and width of the door, you will draw the door, making it of the height shown in the section, or in the former elevation.

From the points 4 and 5, which show the width of the window, you will raise perpendiculars, in order to find the position of the sides of the window.

For the height of the window, and its particular height above the ground line, you will refer to the former elevation; and you will complete the form of the window accordingly.

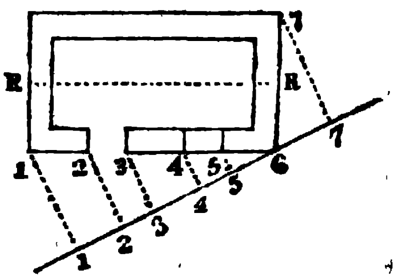


Dot such part of each of the perpendiculars, raised from the points 4 and 5, as falls below the bottom of the window.

The roof only remains to be drawn. Before this can be done, it will be necessary to find the points where the extremities of the ridge of it ought to be laid down in the plane of projection.

The ground plan of the cottage does not show the ridge of the roof; but it must be evident that the ridge of a regular simple pitch roof, if the plan of it were drawn, would fall exactly over the middle of the building.

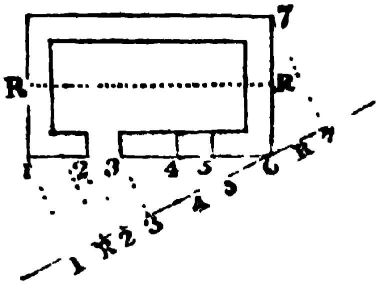
In order to save the trouble of drawing a separate plan of the roof of the cottage, I shall therefore add a dotted line to my ground plan, to represent that part over which the ridge of the roof would fall; and I shall mark the two extremities of this dotted line by the letters R and R, in order to show the extreme points of the said ridge.



You will follow my example by drawing a dotted line in your plans, and marking the extremities of it in the same manner; in doing which you must take care to make it fall exactly over the middle of your building.

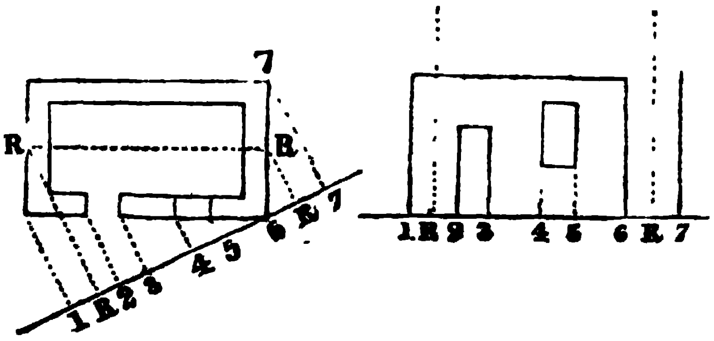
From the points marked R and R, which represent the extremities of the ridge of the roof, you will draw rays, or perpendiculars to the plane of projection, and dot them.

Mark the points where these rays meet the plane of projection, by the same letters R and R, in order to show that they correspond with the extremities of the roof, as laid down in the plan.

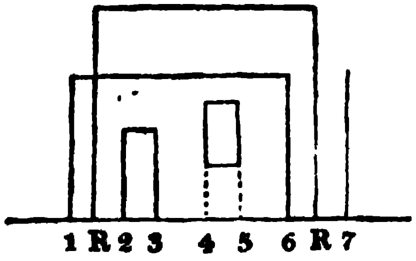


The points R and R, marked on the line which represents the plane of projection, must next be transferred to the ground line of the oblique elevation; upon which you will also mark the new points, thus found, by the corresponding letters R and R.

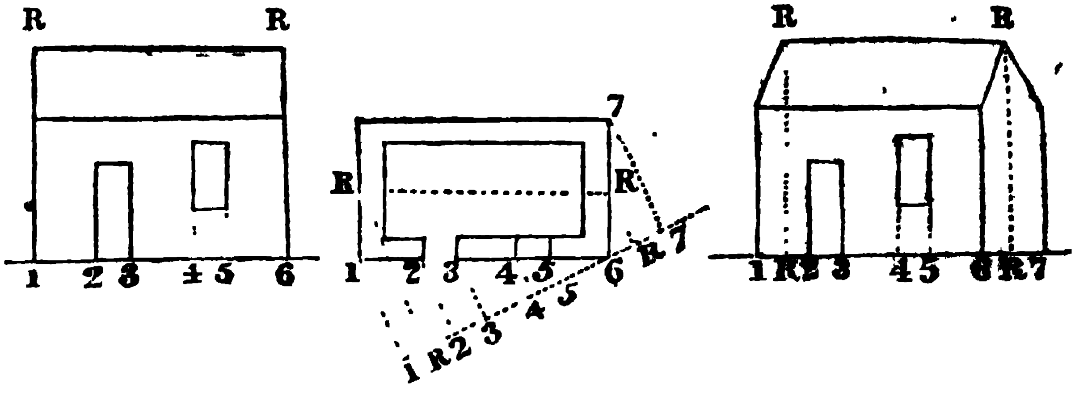
From the new points R and R, on the ground line of your elevation, you will next raise dotted perpendiculars, in order to find the proper position of the extremities of the ridge of the roof.



The ridge or top of the roof must next be drawn, parallel to the ground line, and extending between the two last raised dotted perpendiculars. Its perpendicular height must be found by referring to the section, or to the former elevation.



Draw oblique lines from the extremities of the ridge of the roof, to the proper points, in order to complete the form of the roof, in the elevation.



The oblique elevation of the cottage is now finished. The heights of the various parts of it agree with the heights given in the section and in the former elevation of the front of the cottage; but all the other dimensions are diminished from their true proportions, in which they appear in the plan and in the former elevation.

If the oblique plane of projection, by means of which the last elevation was drawn, had formed a more obtuse angle with the front of the building; then the various dimensions, in that part of the oblique elevation, which represents the front of the cottage, must have been still more diminished than they are at present.

In the proper or direct elevation of the front of the cottage it was not necessary to take any notice of the points R and R, which represent the extremities of the ridge of the roof; because the plane of projection was then supposed to coincide or agree with the front of the building, and from the simple nature of the roof, the points R and R in the plan stand exactly over the two ends of the cottage.

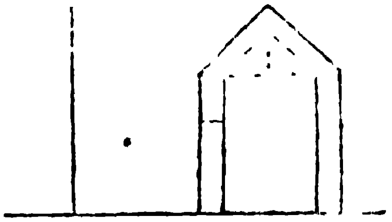
Therefore as the ends are perpendicular to the front of the building, it must be evident, that if rays or perpendiculars had been drawn from the points R and R in the plan, they must have exactly coincided with the points 1 and 6 marked at the extremities of the front of the building, which in the first elevation we drew was supposed to represent the plane of projection; and if the plane of projection had been supposed to be parallel to the front of the building, the same coincidence or agreement between the points R, R, and 1, 6, would still have taken place, for the reasons which I before stated to you, in treating of direct or proper geometrical elevations.*

In transferring the various heights from the section to your two elevations, you measured each dimension separately and individually one after another; but it is also usual to transfer the various heights from a section to an elevation all at once, in the manner in which you transferred your dimensions from the plan to the elevation.

I shall give you an example of the proper method of doing this, which you will find very simple.

The front of the cottage, which is a vertical plane, being represented in the section by a perpendicular right line, any other vertical plane parallel to it must also be represented, in a section, in the same manner.

You will therefore, at any convenient distance, raise a perpendicular parallel to that line in the section which represents the front of the cottage.



The perpendicular just drawn, being a vertical plane parallel to the front of the cottage, may represent a plane of projection supposed to be used for finding the proper proportions of the various parts of the building, in drawing a geometrical elevation of it.

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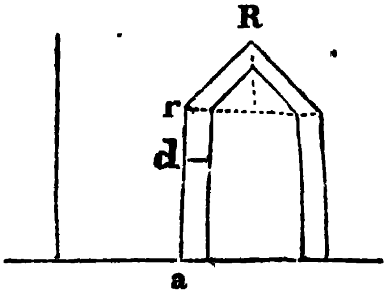
* It will easily be understood, that if the roof were not of the simple form, represented in the figures; that is to say, if it were supposed to have more than two sloping sides, then rays projected from the points R, R, would not agree with the points 1 and 6, even in a proper or direct geometrical elevation.

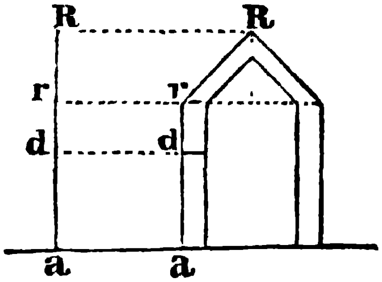
You will mark all those points of the front of the building, shown in the section, which would appear in a geometrical elevation of it, by the letters a, d, r, and R.



From the several points, thus marked, rays must be thrown out, or in other words, perpendiculars must be drawn to the line which represents the plane of projection.

You will dot all of these perpendiculars excepting that which agrees with the ground line or base of the section.

You will, in like manner, mark the points, where these rays meet the plane of projection, by corresponding letters a, d, r, and R.



The distance between the two points marked a and d in the section represents the height of the door of the cottage.

The distance between the two corresponding points a and d, on the plane of projection, will therefore also represent the height which ought to be given to the door, in a geometrical elevation.

The distance between the two points marked a and r, on the section, represents the height of the front wall of the cottage.

The distance between the two corresponding points a and r, on the plane of projection, will therefore also represent the height, which ought to be given to the front wall of the cottage; in a geometrical elevation.

These dimensions are all equal to each other, because the plane of projection and the front wall of the cottage are parallel.

The distance between the points marked r and R, in the section, shows the slope of the front of the roof of the cottage.

The distance between the corresponding points r and R, on the plane of projection, will therefore represent the height, which ought to be given to the roof in a geometrical elevation.

The last mentioned distance is equal to the dotted perpendicular, which you before drew to show the proper height of the roof; and it is therefore less than the oblique length of the slope of the roof, being diminished according to the rules of projection.

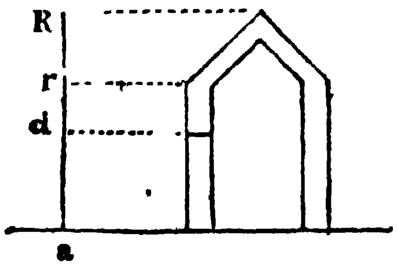
All the points necessary for transferring the several heights of the front of the building from a section, to an elevation, are now marked on the plane of projection.

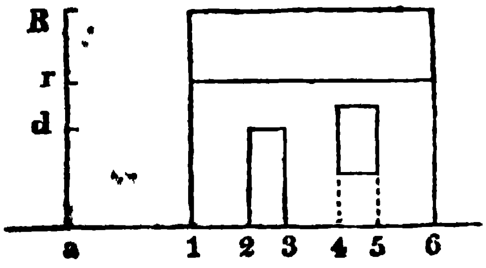
I will next show you in what manner this may be done.

From the ground line of your elevation produced, raise a perpendicular equal in length to the line which represents the plane of projection attached to your section, and let this new perpendicular be divided into the same number of parts, and in the same proportion as the former.

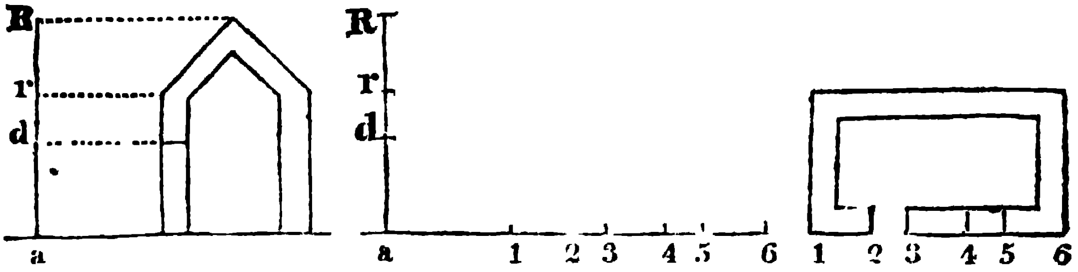
Mark it also by the same letters a, d, r, and R.

The last drawn line will serve as a scale by which the various heights might have been transferred from the section to the elevation. We shall call it the scale of heights.



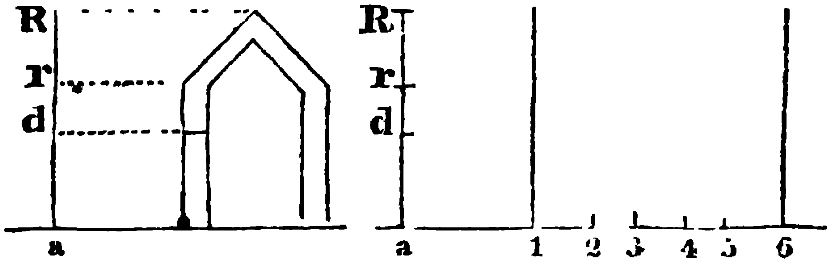


Rub out the whole of your elevation excepting the ground line of it, and the various points 1, 2, 3, 4, 5, and 6, which were marked upon the said ground line, in order to agree with the dimensions of the front of the building, represented in the plan.



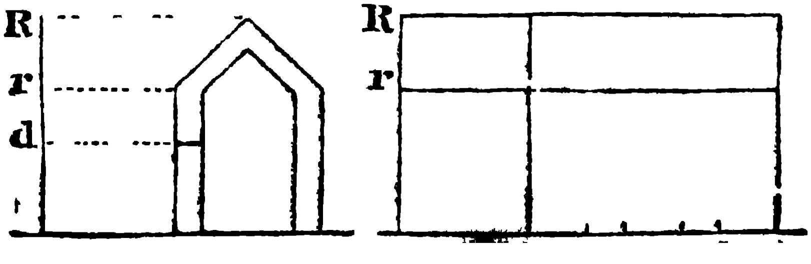
You must now draw your elevation again, by means of the various points marked on the scale of heights, and on the ground line.

From the points 1 and 6 on the ground line of your proposed elevation, which represent the extremities of the front of the cottage, raise perpendiculars to show the height of the walls.



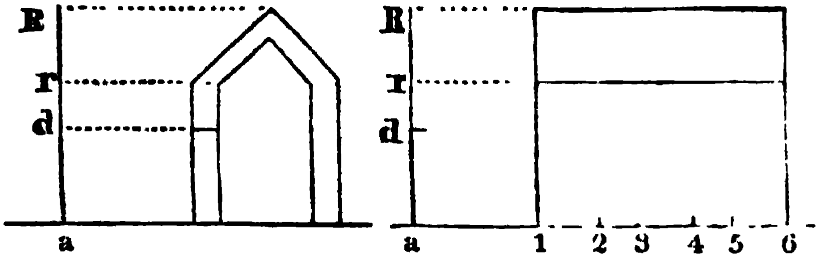
The point marked R on the scale for your elevation, shows the height of the top of the roof, and the point marked r on the same scale shows the height of the bottom of the roof, or of the top of the front wall of your shed, which is the same thing.

From the points R and r on the scale of heights, you will therefore draw lines parallel to each other and to the ground line, intersecting the two perpendiculars, which were raised from the points 1 and 6. These will of course be horizontal lines.



The two last drawn lines will represent the level of the top and bottom of the roof of the building: but the roof cannot extend further than between the two perpendiculars, which are raised from the points 1 and 6, which represent the extremities of the front of the building.

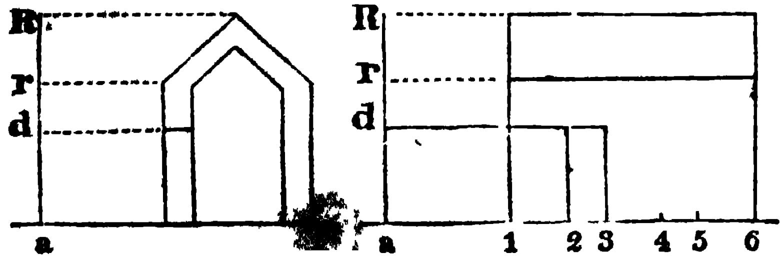
You will therefore dot the superfluous parts of the horizontal lines which are drawn from the points R and r; that is to say, such part of each of them as is not included between the above perpendiculars.



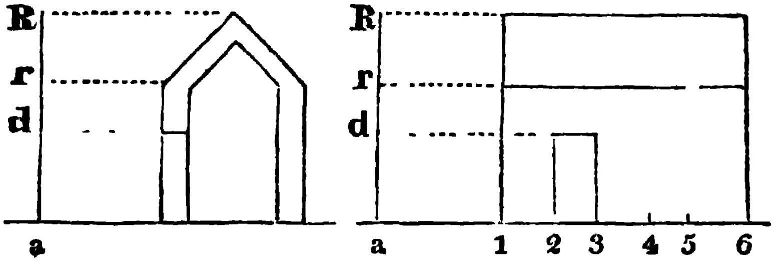
Thus by raising perpendicular or vertical lines from the points 1 and 6 on the ground line of the elevation, which correspond with the extremities of the building; and by drawing parallel or horizontal lines to intersect the former, from the points R and r on the scale of heights, which show the level of the top and bottom of the roof, you have found the general form of the front of the building.

From the points 2 and 3 on the ground line of your elevation, which represent the width of the door, raise perpendiculars for the sides of the door.

From the point d on the scale of heights, which shows the height of the top of the door, draw a horizontal line intersecting the above perpendiculars.



The last drawn line will show the level of the top of the door, but that part of it only which falls within the two perpendiculars, represents the actual form of the top of the door. You will therefore dot the remaining or superfluous parts of the last drawn horizontal line.



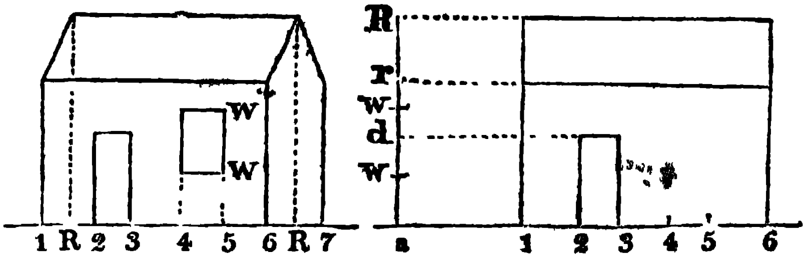
The height of the window is not shown in the section, and therefore could not be transferred from thence to the scale of heights: but as it is represented in your oblique elevation, you may measure it accordingly.

Take in your compasses the height of the top of the window from the ground line of the cottage, as represented in your oblique elevation, and mark a point at the same height above the ground line, upon the scale of heights attached to your unfinished elevation.

Let this point be marked by the letter w.

Take in like manner in your compasses the height of the bottom of the window, as represented in the oblique elevation, and set it off upon the scale of heights attached to your unfinished elevation.

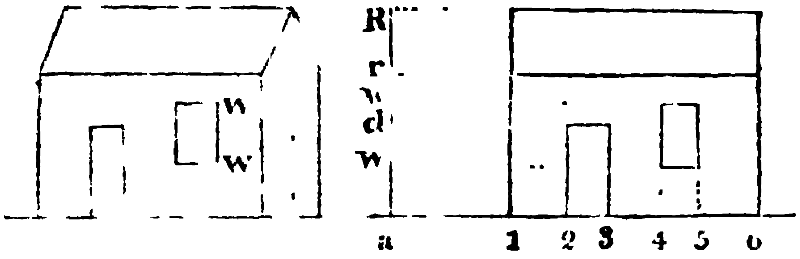
Mark also this new point by the letter w.



The top and bottom of the window in the oblique elevation shall also be marked by the letters w and w to show the manner in which they agree with the former points.

From the points 4 and 5 on the ground line of your elevation, you will raise perpendiculars to find the position of the sides of the window, and from the points w and w on the scale of heights, you will draw horizontal lines, intersecting the former, in order to find the position of the top and bottom of the window.

When by this means you have drawn the window, you will dot the superfluous parts of the above perpendicular and horizontal lines, which assisted you in finding the position of it.



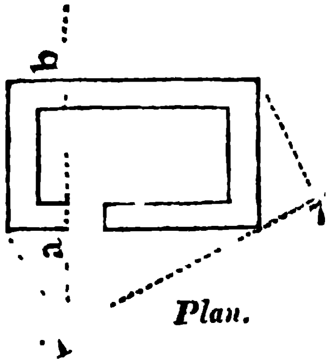
Thus by intersecting lines drawn perpendicular or parallel to the ground line of your elevation, from certain points, you have completed a new direct or proper geometrical elevation equal and similar to that which you before drew.

In this example, as well as in the former, the various points, necessary for drawing the proper elevation of the front of the cottage, were found by supposing a vertical plane of projection to be used, either coinciding with the front of the building, or parallel to it.

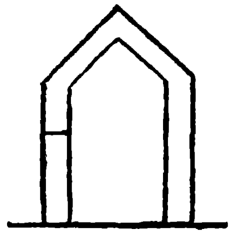
After finishing any elevation, or other geometrical drawing, the superfluous or dotted imaginary lines representing planes of projection, rays, scales of heights, &c. as also the numeral figures, letters, &c. attached to them, would be rubbed out, excepting those imaginary lines only, marked in a plan, which show the direction according to which the sections or oblique elevations, accompanying the plan, may have been taken.

I shall accordingly rub out superfluous lines, letters, &c. in the figures which I have drawn [illegible] the board, relative to the cottage, leaving only such superfluous or imaginary lines, &c. as are necessary for explaining the connection between the plan and section, and between the plan and oblique elevation.

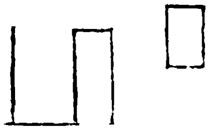
In doing this, I shall dot the line which represents the oblique plane of projection in the plan, because in finished drawings all imaginary lines are dotted.



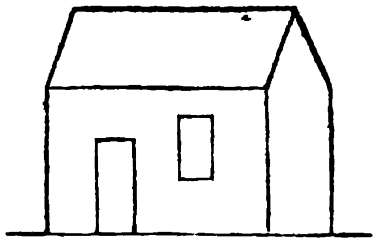
Plan.



Section on the line a b.



Elevation of the Front.



Oblique Elevation on the line 1 7.

You will alter your respective figures in the same manner.

The plan, section, and elevations, which you have now completed, are sufficient to give you a thorough insight into the principles of Plan-drawing.

As an example, I have chosen a building of the most simple nature, in order to lay down the rules of the art in as clear a manner as possible. But the same rules will apply in all other cases; so that if you understand what you have already done, you will find no difficulty in comprehending geometrical drawings of the most extensive and complicated buildings or other objects.

All plans, sections, and elevations, are drawn by laying down a certain number of points and lines truly on some plane surface, according to geometrical principles. In drawing some objects, it may be necessary to lay down a great number of points and lines, in others only a few: but whether the number be great or small, in all cases, each individual point or line must be drawn according to some one or other of the foregoing rules.

Many persons have considerable practice in Plan-drawing, and are able to understand simple plans, sections, and direct elevations, without having been regularly instructed in the principles of the art which have now been explained to you.

If you find any difficulty in understanding such of the above rules, as relate to the planes of projection, which are supposed to be used for laying down oblique, or sloping lines properly, in plans and elevations, do not allow yourselves to be discouraged on that account.

The nature of sections, and all the simpler kind of plans, which are most generally useful in the Royal Engineer department, may be understood without a thorough knowledge of all the rules of projection; and after some practice in drawing or in working from plans and sections, if you return to the study of these rules, and reflect upon them with due attention, they will appear sufficiently simple and easy.

This remark is intended for your encouragement, and guidance, in order that you may not be deterred from attempting to learn the practice of Plan-drawing, in consequence of any difficulty which you may at present meet with in regard to the rules, or principles, of the art.

But although the principles of Plan-drawing may certainly be learned to greater advantage, after some experience in the practical part, it would not have been proper in a regular Course of Instruction, to introduce the practice first. I have therefore commenced by explaining the principles.

Elevations of any building or other object very much resemble the outward appearance of the object represented by them, when viewed by any person standing near it; and therefore geometrical drawings of this description convey an idea of the nature of the object represented, even to persons who may be totally ignorant of the principles of Plan-drawing.

Sections do not convey any just notion of the object represented to ignorant persons, because the section of any object (a building for instance) supposes it to be cut in two, so that the figure or appearance of the object, thus formed, does not resemble any natural or common view of it.

In like manner, plans, which, as was before stated, resemble the appearance of any object viewed from a height immediately above it, do not convey a very just notion of the object to persons ignorant of the principles of Plan-drawing; because opportunities of looking perpendicularly or directly down upon objects, or in other words of taking a bird's-eye view of them, are not common.

Ground plans, or foundation plans of buildings, or other works, do not give any just notion of the appearance of the object represented: because when a building is finished, it is impossible, from any point of view whatever, to see the various walls and foundations in the manner in which they must be represented in a ground plan, the whole of these parts being hidden by the roof. In fact, the ground plan of any finished building, is, properly speaking, a section through the various walls, the only difference between it and a common section consisting in this; that the common section is taken vertically, whereas the actual section of a building, which is exhibited in any ground plan of it, is taken horizontally.

I have already stated that plans, and sections of a very simple nature, are of most general use in the Royal Engineer department. In works of fortification, geometrical elevations of any kind are seldom necessary; but they are highly useful in architecture, because the general appearance of a building cannot be understood, nor many useful dimensions well explained, without them.

Elevations, as I before observed, are the only kind of geometrical drawings which can convey any just notion of the outward appearance of a building or other object, to persons who are ignorant of the principles of Plan-drawing: but although geometrical elevations very much resemble the outward appearance of any thing, they are not an exact and faithful representation of objects as they appear to the eye of a spectator.

For instance, a person can only view an object from one point: therefore any part of a large building, such as a window, which happens to be near his eye, will appear much larger than another window of the same size at some more distant part of the building. This is a remark which every one must have made. But in a geometrical elevation, &c. all those parts of the same object, which are equal to each other in reality, are also made equal to each other in the drawing.

Drawings which represent objects in the same proportions, in which they would appear to the eye, when viewed from any given point or distance, are called perspective drawings:

And a knowledge of the methods according to which the various proportions of objects should be laid down in a drawing of this kind, is called the Art of Perspective.

The Art of Perspective, like that of Plan-drawing, can neither be understood nor put in practice without understanding the principles of Geometry.

Perspective is principally useful and necessary to painters, who proportion the various dimensions of their figures in pictures, views, and landscapes, according to the rules of this art.

Perspective views give a much better notion of the outward appearance of objects than geometrical drawings; but they are not of so much use in the practical branches of mechanics, because all the parts in a perspective drawing being made longer or shorter, than their real proportions, according to their supposed distance from the eye of a spectator; it becomes impossible to ascertain, the true dimensions of any object, which is represented in a drawing of this kind, by means of a scale or by measurement.

In plans, sections and elevations of any object, when the various points and lines have been laid down according to the rules of projection, it is usual afterwards to colour or shade the figure in order to make a finished drawing of it.

The art of Plan-drawing therefore comprehends two distinct operations: first the projection of the lines which form the representation of any object; and secondly the colouring or shading of it.

The Teacher should have some coloured or shaded plans, sections, and elevations, which he ought to exhibit, in order to illustrate the following remarks.

In coloured plans and sections masonry is generally made red; wood so as to resemble its own natural colour; earth of a sandy colour; iron of a dark blue; and water of a lightish blue.

In plans of buildings, not coloured, masonry is generally made dark, whilst wood and other substances are shaded lighter.

In sections, not coloured, different substances are shaded darker or lighter, according to the fancy of the draftsman.

In plans of buildings, the doors and windows are left blank, whilst the walls are either colou d or shaded. And in sections, a marked distinction of shade or colour is also made between the solid part of walls, and the doors, windows, or other apertures, which may be represented.

In elevations of any object, whether coloured or not, the various parts are shaded in such a manner as to resemble the outward appearance of the object, as much as possible.

In plans and elevations, all oblique or sloping planes are generally shaded stronger at one side of the slope than at the other; but the difference of shade must be gradual, that is to say, the dark part must be softened off by degrees.

All curved superficies, whether they are of a regular form, such as the surface of a globe, cylinder, &c. or not, are also shaded off gradually, but in different methods according to the form of the object which is to be represented.

Sometimes, in addition to the common colours and shades, above alluded to, stronger shades are introduced in plans and elevations, in order to represent the shadows, which would be thrown out from all the high or projecting parts of the given object, if the sun were shining upon it, in a certain direction.

The shades introduced into a plan or elevation to represent shadows, are not always softened off like the others, but are in general made equally strong throughout.

When shadows are introduced in geometrical drawings, the light is always supposed to shine from the left extremity of the top of the drawing downwards, but in a diagonal direction, consequently those sides of any object represented which face towards the top or left of the drawing are made light, whilst those that face towards the right side or bottom of the drawing are shaded dark.

I have made these observations, merely for your information, in order to prepare you for the colours and shades which you are likely to meet with, in finished plans, sections, and elevations;

but it is not my intention to enlarge any further upon this branch of the art of Plan-drawing.

A knowledge of the principles of projection, or in other words the art of drawing the outline or figure of any object properly, is by far the most important part of Plan-drawing. If you understand this much, you know almost all that is necessary for every practical purpose.

Colours and shades do not by any means form so essential a branch of the art of Plan-drawing. They serve partly by way of embellishment or ornament; and partly to render a plan, section, or elevation, clearer; which is their principal use. Indeed, so completely subservient and inferior is the art of shading or colouring to that of projection, that the most beautifully finished plan, section, or elevation, would not be of the smallest value, if the outline or figure was wrong.

Shades or colours are more particularly useful in the map of a country, than in any other kind of geometrical drawing. There they are necessary in order to distinguish properly woods, water, marshes, hills, mountains, &c.

A person cannot attain any proficiency in the art of shading and colouring without the assistance of a master, or unless he practises a considerable time in copying from proper books and drawings. But after a little observation of geometrical drawings, you will easily be able to understand the meaning of the various shades and colours, which you may find in them, without any particular instructions on the subject.

END OF THE PRINCIPLES OF PLAN DRAWING,

AND OF

VOLUME FIRST.

J. Innes, Printer,
Wells Street, Oxford Street, London.

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